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Some Results in view of Nevanlinna Theory

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1 Introduction

In this article we deal only with the case of characteristic 0. We recall two Lang's Conjectures:

L.C.1 ('60, '74) Let V be an algebraic variety defined over a number field F. Assume that V with some embedding $F \hookrightarrow \mathbf{C}$ is Kobayashi hyperbolic. Then $|V(F)| < \infty$? Does the analogue over function fields hold, either?

L.C.2 ('66) Let $f : \mathbf{C} \to A$ be an analytic 1-parameter subgroup (or holomorphic curve) in an Abelian variety A, and D be an ample divisor of A. Then $f(\mathbf{C}) \cap D \neq \emptyset$?

For L.C.1 a basic conjectural observation is the correspondence:

1.1 *non-constant holomorphic curves* $f : \mathbf{C} \to V$ $\Longleftrightarrow$ *an infinite set of rational points on* V.

Theorem 1.2 (i) *L.C.1 over function fields holds in the case of* $\dim V = 1$. (Manin [M63], Grauert [Gra65], Noguchi [No85].)

(ii) *L.C.1 over number fields holds in the case of* $\dim V = 1$. (Faltings [Fa83].)

(iii) *L.C.1 over function fields holds in arbitrary* $\dim V \geqq 1$. (Noguchi [No85], [No92].)

(iv) *For a subvariety V of an Abelian variety L.C.1 holds. Moreover, this holds on semi-Abelian varieties in a generalized sense.* (Faltings [Fa91], Vojta [Vo96].)

L.C.1 over number fields is open for $\dim V > 1$. As the Nevanlinna theory is a powerful tool to prove the hyperbolicity of a complex manifold, so is the Diophantine approximation to obtain the finiteness of the set of rational points of a variety. In this context, Vojta [Vo87] made observation 1.1 deeper to the analogue

1.3 *Nevanlinna theory* $\Longleftrightarrow$ *Diophantine approximation.*

2 S.M.T. and integral points

After Vojta's analogue 1.3 Roth-Schmidt's approximation corresponds to the S.M.T. (second main theorem) in Nevanlinna-Cartan theory.

Theorem 2.1 ([Ca33]) *Let $f : \mathbf{C} \to \mathbf{P}^n(\mathbf{C})$ be a linearly non-degenerate holomorphic curve. Let $\{H_j\}_{j=1}^q$ be a finite family of hyperplanes in $\mathbf{P}^n(\mathbf{C})$ in general position. Then*

$$(q-n-1)T_f(r) < \sum_{j=1}^{q} N_n(r, f^*H_j) + O(\log(rT_f(r)))||.$$

Here $T_f(r)$ is the order function of f with respect the hyperplane bundle $O(1)_{\mathbf{P}^n(\mathbf{C})}$ over $\mathbf{P}^n(\mathbf{C})$, and $N_n(r, f^*H_j)$ the counting function of the divisor f^*H_j, truncated at level n (cf. [Ca33], [Fu93]). The truncation for the counting functions are very important and is related to "*abc*-Conjecture" in number theory. An implication of this theorem is

Corollary 2.2 (Borel's Theorem) *Let $f_1(z), f_2(z), \ldots, f_s(z)$ be entire functions which are units (zero free). Assume the following unit equation.*

$$f_1(z) + f_2(z) + \cdots + f_s(z) = 0.$$

Then there is a partition $\{1, \ldots, s\} = \bigcup I_\lambda$ of the index satisfying the following.

(i) $|I_\lambda| \geqq 2$.

(ii) *For arbitrary $i, j \in I_\lambda$, the function $\dfrac{f_i(z)}{f_j(z)} = c_{ij}$ is constant.*

(iii) *For every λ, $\sum_{i \in I_\lambda} f_i(z) = 0$.*

Theorem 2.3 (Roth-Schmidt) *Let F be a number field, and S a finite set of places including all infinite places of F. Let $\{H_j\}_{j=1}^q$ be a finite family of hyperplanes in $\mathbf{P}_F$ in general position. Then for an arbitrary $\epsilon > 0$ there is a finite union E of proper linear subspaces such that for $x \in \mathbf{P}^n(F) \setminus E$*

$$(q-n-1-\epsilon)\mathrm{Ht}(x) < \sum_{j=1}^{q} N(S; H_j(x)) + \mathrm{Const}.$$

An immediate consequence analogous to Corollary 2.2 is

Corollary 2.4 *Let $\mathcal{Z}$ be the set of all S-unit solutions of equation*

$$a_1x_1 + \cdots + a_sx_s = 0 \qquad (s \geqq 2)$$

with $a_j \in F^*$. Then there is a finite decomposition $\mathcal{Z} = \cup_{\mu=1}^{\mu_0} \mathcal{Z}_\mu$ ($\mu_0 < \infty$) such that for every fixed $\mathcal{Z}_\mu$, $1 \leqq \mu \leqq \mu_0$, there is a decomposition of indices $\{1, \ldots, s\} = \bigcup_{l=1}^m I_l$ satisfying the following conditions:

(i) $|I_l| \geqq 2$ *for all* l.

(ii) *If we write* $\mathcal{Z}_\mu = \{(x_i(\zeta)); \zeta \in \mathcal{Z}_\mu\}$ *and take an arbitrarily fixed* I_l, *then*

$$\frac{x_j(\zeta)}{x_k(\zeta)} = c_{jk} \in \mathcal{O}_S^*$$

are independent of $\zeta \in \mathcal{Z}_\mu$ *for all* $j, k \in I_l$.

(iii) $\sum_{j \in I_l} a_j x_j(\zeta) = 0$ *for* $\zeta \in \mathcal{Z}_\mu$ *and* $l = 1, 2, \ldots, m$.

The ways of the arguments to deduce Corollaries 2.2 and 2.4 from Theorems 2.1 and 2.3 are almost identical by making use of the induction on s ([No97]).

Theorem 2.5 (M. Ru and P.-M. Wong [RW91]) *Let* $\{H_j\}_{j=1}^q$ *be hyperplanes in* $\mathbf{P}_F^n$ *in general position. Let* A *be a* $(\sum_{j=1}^q H_j, S)$*-integral point set. Then* A *is contained in a finite union* W *of linear subspaces such that*

$$\dim W \leqq (2n + 1 - q)^+.$$

In special, A *is finite for* $q \geqq 2n + 1$.

In their proof, Nochka's weight was essential. But, in the case of Nevanlinna theory Fujimoto '72 and Green '72 independently obtained an optimal dimension estimate

$$\dim \leqq [n/(q - n)], \qquad q > n,$$

where $[*]$ stands for Gauss' symbol. In fact, we have the same dimension estimate as above for A in Theorem 2.5 in more general context as follows.

Theorem 2.6 ([NW99]) *Let* V *be an* n*-dimensional projective algebraic variety defined over* F. *Let* $\{D_j\}_{j=1}^q$ *be a family of effective divisors on* V_F *in general position.*

(i) *Assume that all* D_i *are ample and that* $q > n(\mathrm{rank}_{\mathbf{Z}} \mathrm{NS}(V) + 1)$. *Then any* $(\sum_{i=1}^l D_i, S)$*-integral point set of* $V(K)$ *is finite.*

(ii) *Let* $X \subset \mathbf{P}_F^m$ *be an irreducible subvariety, and let* $D_j, 1 \leqq j \leqq q$, *be distinct hypersurface cuts of* X *that are in general position as hypersurfaces of* X. *If* $q > 2 \dim X$, *then any* $(\sum_{j=1}^q D_j, S)$*-integral point set of* $X(K)$ *is finite.*

(iii) *Let* $D_j, 1 \leqq j \leqq q$, *be ample divisors of* V *in general position. Let* A *be a subset of* $V(K)$ *such that for every* D_j, *either* $A \subset D_j$, *or* A *is a* $(\sum_{D_j \not\supset A} D_j, S)$*-integral point set. Assume that* $q > n$. *Then* A *is contained in an algebraic subvariety* W *of* V *such that*

$$\dim W \leqq \left[\frac{n}{q-n} \mathrm{rank}_{\mathbf{Z}} \mathrm{NS}(V)\right].$$

In the special case of $V = \mathbf{P}_K^m$ *we have*

$$\dim W \leqq \left[\frac{n}{q-n}\right].$$

In the proof we use Vojta's result ('96), which in the Nevanlinna theory is known as log-Bloch-Ochiai's Theorem ([No77], [No81]).

3 S.M.T. over function fields

As we saw the force of the estimate of S.M.T., it is interesting to deal with it over function fields. Actually a number of people have obtained such estimates (Mason, Voloch, Brownawell-Masser, J. Wang, Noguchi ...). We recall

Theorem 3.1 ([No97]) *Let $x = [x_0, \dots, x_n] : R \to \mathbf{P}^n(\mathbf{C})$ be a morphism from a smooth projective variety (or compact Kähler manifold) R with a Kähler form ω. Let $H_1, \dots, H_q$ $(q \geqq n+1)$ be linear forms on $\mathbf{P}^n(\mathbf{C})$ in general position such that the divisor $(H_j(x))$ is defined for every j. Let r denote the rank of dx at general point, and let l denote the dimension of the smallest linear subspace of $\mathbf{P}^n(\mathbf{C})$ containing $x(R)$. Then*

$$(q - 2n + l - 1)\mathrm{ht}(x;\omega) \leqq \sum_{i=1}^{q} N_{l-r+1}((H_i(x));\omega)$$
$$+ \left\{ \frac{(l-r+1)(l-r+2)}{2} + r - 1 \right\} \frac{2n-l+1}{l+1} N(J;\omega).$$

If $\dim R = 1$, $N(J;\omega) = 2g-2$, where g denotes the genus of R.

Here, we set

$$\mathrm{ht}(x;\omega) = \int_R x^* c_1(O(1)_{\mathbf{P}^n(\mathbf{C})}) \wedge \omega^{\dim R - 1},$$
$$N_{l-r+1}((H_i(x));\omega) = \int_{a \in \{H_j(x)=0\}} \min\{\mathrm{ord}_a(H_j(x)), l-r+1\}\, \omega^{\dim R-1}.$$

J. Wang dealt with the case where the coefficients of $H_i, 1 \leqq i \leqq q$ are not constants, but elements of the function field over R when $\dim R = 1$ (see [W96], [W00]). The case of non-constant coefficients is of interest from the viewpoint of the Diophantine approximation; the variables should belong to the same field as the coefficients'. In the estimate we still need to make effective and clear the following points:

(i) In the proof of her result there was a part to chase the lower-limit

$$\liminf_{r\to\infty} \frac{d(r+1)}{d(r)} = 1,$$

where $d(r)$ denotes the dimension of the vector space generated by the r-th products of coefficients of H_i. This r is involved in the coefficients and the constant terms of the approximation; this is also the case for H_j with constant coefficients.

(ii) The level of the truncation of counting functions is not computed at all.

If the counting functions are not truncated, then the first and second main theorems coincide with Poincaré's duality, or a special case of complete intersection theory over compact varieties. The truncation in counting functions is as essentially important as in the case of "*abc*-Conjecture". We will show some S.M.T. where the truncation level is a bit complicated, but computable.

By making use of Theorem 3.1 and the method of Steinmetz [St85]-Shirosaki [Sh91] we have

Theorem 3.2 *Let* $\dim R = 1$ *and let the genus of* R *be* g. *Let* L *be a line bundle on* R *whose degree is* $\deg L$. *Let* $H_j, 1 \leqq j \leqq q$, *be linear forms on* $\mathbf{P}^n$, *with coefficients* $a_{ji} \in H^0(R, L)$ *such that there is no common zero of* a_{ij}. *For an arbitrarily given* $0 < \epsilon < 1$, *let*

$$p_\epsilon = \max\left\{\left[\frac{2n}{\epsilon}\right] + g, 2g-2\right\} + 1$$

Then for an arbitrary morphism $x : R \to \mathbf{P}^n$, *we have*

$$(q - 2n - \epsilon)\mathrm{ht}(x) \leqq \sum_{j=1}^{q} N_{(n+1)h(p_\epsilon+1)-1}(H_j(x)) + C(\epsilon, q),$$

where $h(p_\epsilon + 1) = (p_\epsilon + 1)\deg L - g + 1$ *and*

$$C(\epsilon, q) = q\binom{q}{n+1}(n+1)(p_\epsilon+1)\deg L + 2n((n+1)h(p_\epsilon+1) - 1)(2g-2)^+.$$

In the proof we use the Riemann-Roch: For $p \deg L > 2g - 2$,

$$h^0(L^p) = p \deg L - g + 1.$$

It immediately follows from Theorem 3.2

Corollary 3.3 *Let the notation be as in Theorem 3.2. Let* $S \subset R$ *be a finite set, and* $q \geqq 2n + 1$. *If* x *is a* $(\sum H_j, S)$*-integral point, i.e., as a mapping* $x^{-1}(\sum H_j) \subset S$, *then*

$$(1 - \epsilon)\mathrm{ht}(x) \leqq q((n+1)h(p_\epsilon+1) - 1)|S| + C(\epsilon, q), \qquad 0 < \epsilon < 1.$$

For the case $\dim R \geqq 2$, we do not know $h^0(L^p)$ so explicitly, but $h^0(L^p)$ is known to be a polynomial of degree at most $\dim R$ for large p. Assuming this polynomial, we can work out the above obtained result for R of $\dim R \geqq 2$. For instance, let $\dim R = 2$. Then, by R.-R.

$$h^0(L^p) - h^1(L^p) + h^2(L^p) = \frac{L \cdot L}{2}p^2 - \frac{L \cdot K_R}{2}p + \chi(\mathcal{O}_R).$$

Assume that L is ample. By the vanishing theorem we see that for all large p (geometrically effective), $h^1(L^p) = h^2(L^p) = 0$, so that

$$h^0(L^p) = \frac{L \cdot L}{2}p^2 - \frac{L \cdot K_R}{2}p + \chi(\mathcal{O}_R).$$

Then for a given $\epsilon > 0$ one can determine effectively p; in all dimensions,

$$p \gtrsim 1/\epsilon.$$

Remark. We explain the meaning of the truncation estimate in relation with *abc*-Conjecture:

$$(1-\epsilon)\log\max\{|a|,|b|,|c|\} \leq \left(\sum_{\text{prime } p} \min\{1, \text{ord}_p a\}\log p \right.$$
$$\left. + \sum_{\text{prime } p} \min\{1, \text{ord}_p b\}\log p + \sum_{\text{prime } p} \min\{1, \text{ord}_p c\}\log p \right) + C_\epsilon,$$

where a, b, c are coprime integers and $a+b+c=0$. Here the truncation level is 1. One may relax the claim so that for some given $k>0$

$$(1-\epsilon)\log\max\{|a|,|b|,|c|\} \leq \left(\sum_{\text{prime } p} \min\{k, \text{ord}_p a\}\log p \right.$$
$$\left. + \sum_{\text{prime } p} \min\{k, \text{ord}_p b\}\log p + \sum_{\text{prime } p} \min\{k, \text{ord}_p c\}\log p \right) + C_\epsilon.$$

Furthermore, one may allow that k depends on ϵ. Unfortunately, the truncation level of N_* in Theorem 3.2 is depending on ϵ, and it is an open problem if it is taken independent of ϵ.

Taking the analogue to Theorem 3.2 one may pose

Little abc-Conjecture. *For $\epsilon>0$, there exit positive constants k_ϵ and C_ϵ such that*

$$(1-\epsilon)\log\max\{|a|,|b|,|c|\} \leq \left(\sum_{\text{prime } p} \min\{k_\epsilon, \text{ord}_p a\}\log p \right.$$
$$\left. + \sum_{\text{prime } p} \min\{k_\epsilon, \text{ord}_p b\}\log p + \sum_{\text{prime } p} \min\{k_\epsilon, \text{ord}_p c\}\log p \right) + C_\epsilon.$$

4 L.C.2

Answering a question raised by S. Lang [L66], Ax [Ax72] proved

Theorem 4.1 *Let $f : \mathbf{C} \to A$ be a non-trivial analytic 1-parameter subgroup of an Abelian variety A, and D be an ample divisor on A. Then*

$$N(r, f^*D) \sim r^2, \qquad r \to \infty.$$

Then Griffiths [Gri74] generalized the question for holomorphic curves which are not necessarily subgroups; then this is a problem of the Nevanlinna theory. Siu-Yeung [SY96] and [No98] proved that non-constant holomorphic f always intersects D.

To deal with more general case we introduce a notion of a semi-torus. Let M be a complex Lie group admitting the exact sequence

$$0 \to (\mathbf{C}^*)^p \to M \xrightarrow{\eta} M_0 \to 0, \tag{4.2}$$

where $\mathbf{C}^*$ is the multiplicative group of non-zero complex numbers, and M_0 is a (compact) complex torus. Such M is called a *complex semi-torus* or a *quasi-torus*. If M_0 is algebraic, that is, an Abelian variety, M is called a *semi-Abelian variety* or a *quasi-Abelian variety*.

Lately we proved

Theorem 4.3 ([NWY99], [NWY00]) *Let $f : \mathbf{C} \to M$ be a holomorphic curve into a complex semi-torus M such that the image $f(\mathbf{C})$ is Zariski dense in M. Let D be an effective divisor on M such that the closure $\bar{D}$ of D in $\bar{M}$ is an effective divisor on $\bar{M}$. Assume that D satisfies a certain boundary condition. Then we have the following.*

(i) *Suppose that f is of finite order ρ_f. Then there is a positive integer $k_0 = k_0(\rho_f, D)$ depending only on ρ_f and D such that*

$$T_f(r; c_1(\bar{D})) = N_{k_0}(r; f^*D) + O(\log r).$$

(ii) *Suppose that f is of infinite order. Then there is a positive integer $k_0 = k_0(f, D)$ depending on f and D such that*

$$T_f(r; c_1(\bar{D})) = N_{k_0}(r; f^*D) + O(\log(rT_f(r; c_1(\bar{D}))))||_E.$$

Cf. [Kr00], [Mc96] and [SY97] for related results. The following very precise estimate is an immediate consequence of Theorem 4.3 (cf. Theorem 4.1):

Theorem 4.4 *Let $f : \mathbf{C} \to A$ be a 1-parameter analytic subgroup in an Abelian variety A with $a = f'(0) \neq 0$. Let D be an effective divisor on A with the Riemann form $H(\cdot, \cdot)$ such that $D \not\supset f(\mathbf{C})$. Then we have*

$$N(r; f^*D) = H(a, a)\pi r^2 + O(\log r).$$

Since $H(a, a) = \lim_{r\to\infty} N(r; f^*D)/\pi r^2$, the Riemann form H may be recovered by the counting functions $N(r; f^*D)$ for 1-parameter analytic subgroups.

Yamanoi [Ya01] proved very lately that

Theorem 4.5 *Let the notation be as above. For arbitrary $\epsilon > 0$*

$$N(r, f^*D) - N_1(r, f^*D) < \epsilon T_f(r, c_1(D))||_\epsilon.$$

Hence we have

$$(1 - \epsilon)T_f(r, c_1(D)) < N_1(r, f^*D)||_\epsilon.$$

Remark 4.6 In the above estimate the term $\epsilon T_f(r, c_1(D))$ cannot be replaced by

$$O(\log(rT_f(r; c_1(\bar{D}))))||_E.$$

In fact, let $E = \mathbf{C}/(\mathbf{Z} + i\mathbf{Z})$ be an elliptic curve, and let D be an irreducible divisor on E^2 with cusp of order l at $0 \in E^2$. Let $f : z \in \mathbf{C} \to (z, z^2) \in E^2$. Then $f(\mathbf{C})$ is Zariski dense in E^2, and

$$T_f(r; L(D)) \sim r^4(1 + o(1)).$$

Note that $f^{-1}(0) = \mathbf{Z} + i\mathbf{Z}$ and $f^*D \geqq N(\mathbf{Z} + i\mathbf{Z})$. For an arbitrary fixed k_0, we take $N > k_0$, and then have

$$N(r; f^*D) - N_{k_0}(r; f^*D) \geqq (N - k_0)r^2(1 + o(1)).$$

The above left-hand side cannot be bounded by $O(\log r)$. This gives also a counter-example to [Kr91], Lemma 4.

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