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by

Li Shumin

UNIVERSITY OF TOKYO

GRADUATE SCHOOL OF MATHEMATICAL SCIENCES

KOMABA, TOKYO, JAPAN

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Li Shumin*

Abstract. In this paper, we consider Maxwell's equations in a biisotropic and inhomogeneous medium. We discuss an inverse problem of determining the coefficients ϵ, ζ, μ in the constitutive relations from a finite number of interior measurements. The proof is done by a $H^{-1}-$Carleman estimate.

Key words. inverse problem, Maxwell's equations, biisotropic, magneto-electric, H^{-1}-Carleman estimate, conditional stability

AMS subject classifications. 15A29, 35Q60

1 Introduction and main results

We consider Maxwell's equations in a biisotropic and inhomogeneous medium:

$$\begin{cases} \partial_t D(x,t) - \nabla \times H(x,t) = 0, \qquad x \in \Omega, \quad -T < t < T, \\ \partial_t B(x,t) + \nabla \times E(x,t) = 0, \qquad x \in \Omega, \quad -T < t < T, \\ \nabla \cdot D(x,t) = \nabla \cdot B(x,t) = 0, \qquad x \in \Omega, \quad -T < t < T, \\ D(x,0) = d(x), \quad B(x,0) = b(x), \qquad x \in \Omega, \\ \nu(x) \times E(x,t) = p(x,t), \qquad x \in \partial\Omega, \quad -T < t < T, \end{cases} \tag{1.1}$$

*Graduate School of Mathematical Sciences, The University of Tokyo, 3-8-1 Komaba Meguro Tokyo 153, Japan (lism@ms.u-tokyo.ac.jp)

with the constitutive relations

$$\begin{cases} D(x,t) = \epsilon(x)E(x,t) + \zeta(x)H(x,t), & x \in \Omega, \quad -T < t < T, \\ B(x,t) = \zeta(x)E(x,t) + \mu(x)H(x,t), & x \in \Omega, \quad -T < t < T. \end{cases} \tag{1.2}$$

Here and henceforth $x = (x_1, x_2, x_3) \in \mathbb{R}^3$, $\partial_t = \frac{\partial}{\partial t}$, $\partial_k = \frac{\partial}{\partial x_k}$ for $k = 1, 2, 3$, $\nabla = (\partial_1, \partial_2, \partial_3)^T$, Δ is the Laplacian in x, Ω is a bounded convex domain in $\mathbb{R}^3$ with the bound $\partial\Omega \subset C^2$, $0 \notin \overline{\Omega}$. $\nu(x) = (\nu_1(x), \nu_2(x), \nu_3(x))^T$ is the outward unit normal vector to $\partial\Omega$ at x. In (1.1),

$$\begin{aligned} D(x,t) &= (D_1(x,t), D_2(x,t), D_3(x,t))^T : && \text{the electric flux density,} \\ B(x,t) &= (B_1(x,t), B_2(x,t), B_3(x,t))^T : && \text{the magnetic flux density,} \\ E(x,t) &= (E_1(x,t), E_2(x,t), E_3(x,t))^T : && \text{the electric field,} \\ H(x,t) &= (H_1(x,t), H_2(x,t), H_3(x,t))^T : && \text{the magnetic field,} \end{aligned}$$

and $d(x)$, $b(x)$, $p(x,t)$ are given vector-value functions, $\epsilon(x)$, $\zeta(x)$ and $\mu(x)$ are scalar functions. Here and henceforth $\cdot^T$ denotes the transpose of vectors or matrices under the consideration.

Our consideration is based on some physical background. In fact, there exist materials which can exhibit the magneto-electric effect. For example, some magnetic crystals such as antiferromagnetic Cr_2O_3 and ferromagnetic $GaFeO_3$ (cf. [19], [16]). For details, we refer to [19], [3], [16] and [17]. The constitutive relations for magneto-electric media can be written in the following form (cf. [19], [3]):

$$\begin{cases} D = \overline{\epsilon}E + \overline{\zeta}H, \\ B = \overline{\zeta}^T E + \overline{\mu}H, \end{cases}$$

where the three 3×3 matrices $\overline{\epsilon}$, $\overline{\mu}$ and $\overline{\zeta}$ are the familiar permittivity, permeability tensors and the Dzyaloshinskii magneto-electric tensor respectively. This paper is concerned with the biisotropic case, that is, $\overline{\epsilon} = \epsilon\mathbf{I}_3$, $\overline{\zeta} = \zeta\mathbf{I}_3$ and $\overline{\mu} = \mu\mathbf{I}_3$ where ϵ, ζ and μ are scalar functions of x and $\mathbf{I}_3$ denotes the 3×3 unit matrix.

In this paper, we consider

Inverse problem: Let $\omega \subset \Omega$ satisfy $\partial\Omega \subset \partial\omega$ and $T > 0$ be suitably given. We consider an inverse problem of determining $\epsilon(x)$, $\zeta(x)$, $\mu(x)$ for $x \in \Omega$ from the observation data

$$D(x,t), \quad B(x,t), \qquad x \in \omega, \quad -T < t < T.$$

For this inverse problem, we will reduce (1.1) with (1.2) to a system composed by equations similar to scalar hyperbolic ones and apply an $H^{-1}-$Carleman estimate to those equations. The method of applying Carleman estimate (i.e., a weighted L^2-estimate) to inverse problems is invented by Bukhgeim and Klibanov [2]. For developments of this method, we refer to Bukhgeim [1], Imanuvilov and Yammamoto [8, 9], Isakov [11, 12], Khaĭdarov [13, 14], Klibanov [15], Yamamoto [24]. For Carleman estimate, we refer to Hörmander [4, 5], Isakov [12]. Imanuvilov [6] proves a new type of Carleman estimate in which the right hand side is estimated in a weighted $H^{-1}-$space and Imanuvilov, Isakov and Yamamoto [7] give another, shorter and independent derivation of an $H^{-1}-$Carleman estimate, which we will use in this paper. Concerning the application of the $H^{-1}-$Carleman estimate to other inverse hyperbolic problems, we refer to Imanuvilov and Yammamoto [10] and Imanuvilov, Isakov and Yamamoto [7]. Furthermore, for other inverse problems for Maxwell's equations, we refer to Romanov [20], Romanov and Kabanikhin [21], Yamamoto [22, 23] and Li and Yamamoto [18].

To state our main results, we introduce some notation. Let $\lambda = \inf_{x\in\Omega} |x|$ and $\Lambda = \sup_{x\in\Omega} |x|$. We assume that

$$\Lambda^2 < 2\lambda^2. \tag{1.3}$$

Let $\mathcal{U} = \mathcal{U}_{\beta,M,\theta_0,\theta_1,\epsilon_0,\zeta_0,\mu_0} =\{(\epsilon,\zeta,\mu) \in \{C^2\left(\overline{\Omega}\right)\}^3$: $\epsilon = \epsilon_0$, $\zeta = \zeta_0$, $\mu = \mu_0$ on $\partial\Omega$; $\|\epsilon\|_{C^2(\overline{\Omega})}$, $\|\zeta\|_{C^2(\overline{\Omega})}$, $\|\mu\|_{C^2(\overline{\Omega})} \le M$; $\epsilon(x)$, $\mu(x)$, $\epsilon(x)\mu(x) - \zeta^2(x) \ge \theta_1$ on $\overline{\Omega}$; $\frac{(\nabla(\epsilon(x)\mu(x)-\zeta^2(x)))\cdot x}{2(\epsilon(x)\mu(x)-\zeta^2(x))} > -\theta_0$ on $\overline{\Omega}$; $2\lambda\beta\left|\nabla\left(\sqrt{\epsilon(x)\mu(x)-\zeta^2(x)}\right)\right| + \beta^2(\epsilon(x)\mu(x)-\zeta^2(x)) < 1-\theta_0$ on $\overline{\Omega}\}$ where the constants $M > 0$, $\theta_0 < 1$, $\theta_1 > 0$, $\beta > 0$ and smooth functions ϵ_0, ζ_0 and μ_0 are suitably given. We let $D[\epsilon,\zeta,\mu;d,b,p](x,t)$, $B[\epsilon,\zeta,\mu;d,b,p](x,t)$, $E[\epsilon,\zeta,\mu;d,b,p](x,t)$ and

$H[\epsilon, \zeta, \mu; d, b, p](x,t)$ satisfy (1.1) and (1.2).

Moreover, for any $W = (w_1, \ldots, w_3)^T$, we set $|W|^2 = \sum_{k=1}^3 |w_k|^2$. Furthermore, $L^2(\Omega)$, $H^1(\omega \times (-T,T))$, etc. denote usual Sobolev spaces.

We will take two sets of the initial and boundary data denoted by

$$
\begin{aligned}
&d^j(x) = \left(d_1^j(x), d_2^j(x), d_3^j(x)\right)^T, \quad b^j(x) = \left(b_1^j(x), b_2^j(x), b_3^j(x)\right)^T, \qquad x \in \Omega, \\
&p^j(x,t) = \left(p_1^j(x,t), p_2^j(x,t), p_3^j(x,t)\right)^T, \qquad x \in \partial\Omega, \quad -T < t < T,
\end{aligned}
$$

where $j = 1, 2$ respectively. For the sake of convenience, we assume that d^j, b^j and p^j $(j = 1,2)$ are sufficiently smooth and that they satisfy sufficient compatibility conditions respectively. Denote by G the 12×9 matrix

$$
\begin{pmatrix}
0 & e_1 \times d^1 & e_1 \times b^1 & 0 & e_2 \times d^1 & e_2 \times b^1 & 0 & e_3 \times d^1 & e_3 \times b^1 \\
e_1 \times d^1 & e_1 \times b^1 & 0 & e_2 \times d^1 & e_2 \times b^1 & 0 & e_3 \times d^1 & e_3 \times b^1 & 0 \\
0 & e_1 \times d^2 & e_1 \times b^2 & 0 & e_2 \times d^2 & e_2 \times b^2 & 0 & e_3 \times d^2 & e_3 \times b^2 \\
e_1 \times d^2 & e_1 \times^2 & 0 & e_2 \times d^2 & e_2 \times b^2 & 0 & e_3 \times d^2 & e_3 \times b^2 & 0
\end{pmatrix}
$$

where $e_1 = (1,0,0)^T$, $e_2 = (0,1,0)^T$ and $e_3 = (0,0,1)^T$.

The following is our main result.

Theorem 1 (Conditional stability). Let the domain Ω satisfy (1.3) and

$$\frac{\Lambda^2 - \lambda^2}{\beta^2} < T^2. \tag{1.4}$$

We assume that there exists a constant $\theta_2 > 0$ such that

$$\text{the deterninant of one of } 9 \times 9 \text{ minors of } G \geq \theta_2, \quad \text{for all } x \in \overline{\Omega}. \tag{1.5}$$

Moreover, we assume that (ϵ, ζ, μ), $(\widetilde{\epsilon}, \widetilde{\zeta}, \widetilde{\mu}) \in \mathcal{U}_{\beta, M, \theta_0, \theta_1, \epsilon_0, \zeta_0, \mu_0}$ and that $D[\epsilon, \zeta, \mu; d^j, b^j, p^j]$, $B[\epsilon, \zeta, \mu; d^j, b^j, p^j]$, $D[\widetilde{\epsilon}, \widetilde{\zeta}, \widetilde{\mu}; d^j, b^j, p^j]$, $B[\widetilde{\epsilon}, \widetilde{\zeta}, \widetilde{\mu}; d^j, b^j, p^j] \in \left(C^3\left(\overline{\Omega \times (-T,T)}\right)\right)^3 (j = 1, 2)$.

Then there are constants $\kappa \in (0,1)$ and $C > 0$ such that

$$\begin{aligned}
&\|\epsilon - \widetilde{\epsilon}\|_{L^2(\Omega)} + \left\|\zeta - \widetilde{\zeta}\right\|_{L^2(\Omega)} + \|\mu - \widetilde{\mu}\|_{L^2(\Omega)} \\
&\le C \left(\sum_{j=1}^{2} \left(\left\|\partial_t \left(D[\epsilon,\zeta,\mu; d^j, b^j, p^j] - D[\widetilde{\epsilon},\widetilde{\zeta},\widetilde{\mu}; d^j, b^j, p^j] \right) \right\|_{(H^1(\omega\times(-T,T)))^3} \right.\right. \\
&\left.\left. + \left\|\partial_t \left(B[\epsilon,\zeta,\mu; d^j, b^j, p^j] - B[\widetilde{\epsilon},\widetilde{\zeta},\widetilde{\mu}; d^j, b^j, p^j] \right) \right\|_{(H^1(\omega\times(-T,T)))^3} \right) \right)^{\kappa}.
\end{aligned} \tag{1.6}$$

Remark 1.1. The initial data satisfying (1.5) exists. For example, we take $d^1(x) = e_3$, $b^1(x) = d^2(x) = e_2$, $b^2(x) = e_1$ for $x \in \overline{\Omega}$. In fact, the 9×9 minor formed by rows 1, 2, 3, 4, 5, 9, 10, 11 and 12 satisfies (1.5) if we take $0 < \theta_2 < 1$.

Remark 1.2. The conditions of (1.3), (1.4) and (ϵ,ζ,μ), $(\widetilde{\epsilon},\widetilde{\zeta},\widetilde{\mu}) \in \mathcal{U}_{\beta,M,\theta_0,\theta_1,\epsilon_0,\zeta_0,\mu_0}$ correspond to those in [7] (i.e., (2.1)-(2.4) and (2.8) in [7]). For more consideration to these conditions, we refer to p.1371 in [7].

Remark 1.3. By settling

$$\mathbb{A}_0 = \begin{pmatrix} \epsilon\mathbf{I}_3 & \zeta\mathbf{I}_3 \\ \zeta\mathbf{I}_3 & \mu\mathbf{I}_3 \end{pmatrix}, \quad \mathbb{A}_k = \begin{pmatrix} 0 & \mathbf{A}_k \\ -\mathbf{A}_k & 0 \end{pmatrix}, \quad k = 1,2,3,$$

$$\mathbf{A}_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \quad \mathbf{A}_2 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \mathbf{A}_3 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

and $\mathbf{U} = (E_1, E_2, E_3, H_1, H_2, H_3)^T$, Maxwell's equations with the constitutive relations (1.2) can be written as

$$\mathbb{A}_0 \partial_t \mathbf{U} + \sum_{k=1}^{3} \mathbb{A}_k \partial_k \mathbf{U} = 0.$$

It is obvious that $\mathbb{A}_0^T = \mathbb{A}_0$ and $\mathbb{A}_k^T = \mathbb{A}_k$ $(k = 1,2,3)$. Moreover, $\mathbb{A}_0$ is a 6×6 positive definite matrix if there exists a constant θ_1 such that ϵ, μ, $\epsilon\mu - \zeta^2 \ge \theta_1$. Therefore, for the direct problem of (1.1) and (1.2), we can refer to the results on symmetric hyperbolic equations.

This paper consists of three sections. In section 2, we will introduce two Carleman estimates, respectively, for a second order hyperbolic equation and a first-order differential equation. In section 3, we will give the proof of theorem 1.

2 Carleman Estimate

For β and λ, we define the functions $\varphi = \varphi(x,t)$ by

$$\varphi(x,t) = \mathrm{e}^{\sigma(|x|^2 - \beta^2 t^2 - \lambda^2)} \tag{2.1}$$

with some large $\sigma > 0$. By noting (1.3) and (1.4), we can assume that $T^2 < \frac{\lambda^2}{\beta^2}$.

Proposition 2.1. Let $\varphi(x,t)$ be given by (2.1). We assume that $(\epsilon, \zeta, \mu) \in \mathcal{U}_{\beta, M, \theta_0, \theta_1, \epsilon_0, \zeta_0, \mu_0}$. Let $u \in H_0^2(\Omega \times (-T,T))$ satisfy

$$(\epsilon(x)\mu(x) - \zeta^2(x))(\partial_t^2 u(x,t)) - \Delta u(x,t) = \widetilde{g} + \partial_t g_0 + \sum_{k=1}^{3} \partial_k g_k, \ x \in \Omega, \ -T < t < T.$$

Then there is $K_1 > 0$ such that for all $s > K_1$

$$\int_{-T}^{T} \int_{\Omega} s|u|^2 \mathrm{e}^{2s\varphi} \mathrm{d}x\mathrm{d}t \leq K_1 \int_{-T}^{T} \int_{\Omega} \left(\frac{1}{s^2} |\widetilde{g}|^2 + \sum_{k=0}^{3} |g_k|^2 \right) \mathrm{e}^{2s\varphi} \mathrm{d}x\mathrm{d}t.$$

Proposition 2.2. Let $\varphi(x,t)$ be given by (2.1). Then there exists $K_2 > 0$ such that for $s > K_2$ we have

$$\int_{\Omega} s|w|^2 \mathrm{e}^{2s\varphi(x,0)} \mathrm{d}x \leq K_2 \int_{\Omega} |\nabla w|^2 \, \mathrm{e}^{2s\varphi(x,0)} \mathrm{d}x$$

for all $w \in C_0^1\left(\overline{\Omega}\right)$.

For the proof of proposition 2.1 and 2.2, we refer to theorem 3.2 and lemma 3.6 in [7] respectively and note that the weight function we use here coincides with that in [7] (cf. (4.3) in [7]).

3 Proof Of Theorem 1

Let

$$\begin{cases} \widehat{D}(x,t;j) = D[\epsilon,\zeta,\mu;d^j,b^j,p^j](x,t), & \widehat{B}(x,t;j) = B[\epsilon,\zeta,\mu;d^j,b^j,p^j](x,t), \\ \widehat{E}(x,t;j) = E[\epsilon,\zeta,\mu;d^j,b^j,p^j](x,t), & \widehat{H}(x,t;j) = H[\epsilon,\zeta,\mu;d^j,b^j,p^j](x,t), \\ \widetilde{D}(x,t;j) = D[\widetilde{\epsilon},\widetilde{\zeta},\widetilde{\mu};d^j,b^j,p^j](x,t), & \widetilde{B}(x,t;j) = B[\widetilde{\epsilon},\widetilde{\zeta},\widetilde{\mu};d^j,b^j,p^j](x,t), \\ \widetilde{E}(x,t;j) = E[\widetilde{\epsilon},\widetilde{\zeta},\widetilde{\mu};d^j,b^j,p^j](x,t), & \widetilde{H}(x,t;j) = H[\widetilde{\epsilon},\widetilde{\zeta},\widetilde{\mu};d^j,b^j,p^j](x,t), \end{cases} \tag{3.1}$$

for $j=1,2$, $x\in\Omega$, $-T<t<T$. By $D[\epsilon,\zeta,\mu;d^j,b^j,p^j]$, $B[\epsilon,\zeta,\mu;d^j,b^j,p^j]$, $D[\widetilde{\epsilon},\ \widetilde{\zeta},\ \widetilde{\mu};\ d^j,$ $b^j,\ p^j]$, $B[\widetilde{\epsilon},\widetilde{\zeta},\widetilde{\mu};d^j,b^j,p^j]\in\left(C^3\left(\overline{\Omega\times(-T,T)}\right)\right)^3 (j=1,2)$, we have

$$\widehat{D}(x,t;j),\ \widehat{B}(x,t;j),\ \widetilde{D}(x,t;j),\ \widetilde{B}(x,t;j)\in\left(C^3\left(\overline{\Omega\times(-T,T)}\right)\right)^3. \tag{3.2}$$

Moreover, by (ϵ,ζ,μ), $(\widetilde{\epsilon},\widetilde{\zeta},\widetilde{\mu})\in\mathcal{U}_{\beta,M,\theta_0,\theta_1,\epsilon_0,\zeta_0,\mu_0}$ and (1.2), it is easy to see that

$$\begin{cases} \widehat{E}(x,t;j) = \gamma_1(x)\widehat{D}(x,t;j)+\gamma_2(x)\widehat{B}(x,t;j), \\ \widehat{H}(x,t;j) = \gamma_2(x)\widehat{D}(x,t;j)+\gamma_3(x)\widehat{B}(x,t;j), \end{cases}$$
$$\begin{cases} \widetilde{E}(x,t;j) = \widetilde{\gamma}_1(x)\widetilde{D}(x,t;j)+\widetilde{\gamma}_2(x)\widetilde{B}(x,t;j), \\ \widetilde{H}(x,t;j) = \widetilde{\gamma}_2(x)\widetilde{D}(x,t;j)+\widetilde{\gamma}_3(x)\widetilde{B}(x,t;j), \end{cases} \tag{3.3}$$

where $x\in\Omega$, $-T<t<T$ and

$$\begin{cases} \gamma_1(x) = \frac{\mu(x)}{\epsilon(x)\mu(x)-\zeta^2(x)}, \\ \gamma_2(x) = -\frac{\zeta(x)}{\epsilon(x)\mu(x)-\zeta^2(x)}, \\ \gamma_3(x) = \frac{\epsilon(x)}{\epsilon(x)\mu(x)-\zeta^2(x)}, \end{cases} \quad \begin{cases} \widetilde{\gamma}_1(x) = \frac{\widetilde{\mu}(x)}{\widetilde{\epsilon}(x)\widetilde{\mu}(x)-\widetilde{\zeta}^2(x)}, \\ \widetilde{\gamma}_2(x) = -\frac{\widetilde{\zeta}(x)}{\widetilde{\epsilon}(x)\widetilde{\mu}(x)-\widetilde{\zeta}^2(x)}, \\ \widetilde{\gamma}_3(x) = \frac{\widetilde{\epsilon}(x)}{\widetilde{\epsilon}(x)\widetilde{\mu}(x)-\widetilde{\zeta}^2(x)}. \end{cases} \tag{3.4}$$

It is obvious that

$$\gamma_1(x)\gamma_3(x)-\gamma_2^2(x) = \frac{1}{\epsilon(x)\mu(x)-\zeta^2(x)}, \quad x\in\overline{\Omega}. \tag{3.5}$$

For $x\in\Omega$, $-T<t<T$, we set

$$f_k(x) = \widetilde{\gamma}_k(x)-\gamma_k(x), \quad k=1,2,3, \tag{3.6}$$

and

$$\begin{cases} Y(x,t;j) = \partial_t \widehat{D}(x,t;j) - \partial_t \widetilde{D}(x,t;j) \in \left(C^2\left(\Omega \times (-T,T)\right)\right)^3, \\ Z(x,t;j) = \partial_t \widehat{B}(x,t;j) - \partial_t \widetilde{B}(x,t;j) \in \left(C^2\left(\Omega \times (-T,T)\right)\right)^3. \end{cases} \tag{3.7}$$

Then we can obtain that, for $x \in \Omega$ and $-T < t < T$,

$$\nabla \cdot Y(x,t;j) = \nabla \cdot Z(x,t;j) = 0, \tag{3.8}$$

$$\partial_t Y(x,t;j) = \nabla \times \left(\gamma_2(x) Y(x,t;j) + \gamma_3(x) Z(x,t;j)\right) - \Psi_2(x,t;j), \tag{3.9}$$

$$\partial_t Z(x,t;j) = -\nabla \times \left(\gamma_1(x) Y(x,t;j) + \gamma_2(x) Z(x,t;j)\right) + \Psi_1(x,t;j), \tag{3.10}$$

$$\begin{aligned} &\xi(x)\left(\partial_t^2 Y(x,t;j)\right) - \Delta Y(x,t;j) = \Phi_1\left(Y(x,t;j), Z(x,t;j)\right) \\ &+\zeta(x)\left(\nabla \times \Psi_2(x,t;j)\right) + \epsilon(x)\left(\nabla \times \Psi_1(x,t;j)\right) - \xi(x)\left(\partial_t \Psi_2(x,t;j)\right), \end{aligned} \tag{3.11}$$

$$\begin{aligned} &\xi(x)\left(\partial_t^2 Z(x,t;j)\right) - \Delta Z(x,t;j) = \Phi_2\left(Y(x,t;j), Z(x,t;j)\right) \\ &+\zeta(x)\left(\nabla \times \Psi_1(x,t;j)\right) + \mu(x)\left(\nabla \times \Psi_2(x,t;j)\right) + \xi(x)\left(\partial_t \Psi_1(x,t;j)\right), \end{aligned} \tag{3.12}$$

where

$$\Psi_k(x,t;j) = \nabla \times \left(f_k(x)\left(\partial_t \widetilde{D}(x,t;j)\right) + f_{k+1}(x)\left(\partial_t \widetilde{B}(x,t;j)\right)\right), k = 1, 2, \tag{3.13}$$

$$\xi(x) = \epsilon(x)\mu(x) - \zeta^2(x), \tag{3.14}$$

$$\begin{aligned} &\Phi_1\left(Y(x,t;j), Z(x,t;j)\right) \\ &= -\zeta(x)\left[(\nabla\gamma_2(x)) \times (\nabla \times Y(x,t;j)) + (\nabla\gamma_3(x)) \times (\nabla \times Z(x,t;j))\right. \\ &\quad \left.+\nabla \times \left((\nabla\gamma_2(x)) \times Y(x,t;j) + (\nabla\gamma_3(x)) \times Z(x,t;j)\right)\right] \\ &\quad -\epsilon(x)\left[(\nabla\gamma_1(x)) \times (\nabla \times Y(x,t;j)) + (\nabla\gamma_2(x)) \times (\nabla \times Z(x,t;j))\right. \\ &\quad \left.+\nabla \times \left((\nabla\gamma_1(x)) \times Y(x,t;j) + (\nabla\gamma_2(x)) \times Z(x,t;j)\right)\right] \\ &\quad +\xi(x)\partial_t\left[(\nabla\gamma_2(x)) \times Y(x,t;j) + (\nabla\gamma_3(x)) \times Z(x,t;j)\right], \end{aligned} \tag{3.15}$$

$$\begin{aligned} &\Phi_2\left(Y(x,t;j), Z(x,t;j)\right) \\ &= -\zeta(x)\left[(\nabla\gamma_1(x)) \times (\nabla \times Y(x,t;j)) + (\nabla\gamma_2(x)) \times (\nabla \times Z(x,t;j))\right. \\ &\quad \left.+\nabla \times \left((\nabla\gamma_1(x)) \times Y(x,t;j) + (\nabla\gamma_2(x)) \times Z(x,t;j)\right)\right] \\ &\quad -\mu(x)\left[(\nabla\gamma_2(x)) \times (\nabla \times Y(x,t;j)) + (\nabla\gamma_3(x)) \times (\nabla \times Z(x,t;j))\right. \\ &\quad \left.+\nabla \times \left((\nabla\gamma_2(x)) \times Y(x,t;j) + (\nabla\gamma_3(x)) \times Z(x,t;j)\right)\right] \\ &\quad -\xi(x)\partial_t\left[(\nabla\gamma_1(x)) \times Y(x,t;j) + (\nabla\gamma_2(x)) \times Z(x,t;j)\right]. \end{aligned} \tag{3.16}$$

In fact, (3.8) is obviously true by noting (1.1), (3.1) and (3.7). By (1.1), (3.3)-(3.4) and (3.6), we have

$$
\begin{aligned}
&Y(x,t;j) = \partial_t \widehat{D}(x,t;j) - \partial_t \widetilde{D}(x,t;j) = \nabla \times \left(\widehat{H}(x,t;j) - \widetilde{H}(x,t;j) \right) \\
&= \nabla \times \left(\gamma_2(x) \left(\widehat{D}(x,t;j) - \widetilde{D}(x,t;j) \right) + \gamma_3(x) \left(\widehat{B}(x,t;j) - \widetilde{B}(x,t;j) \right) \right. \\
&\quad \left. - f_2(x) \widetilde{D}(x,t;j) - f_3(x) \widetilde{B}(x,t;j) \right)
\end{aligned} \tag{3.17}
$$

and

$$
\begin{aligned}
&Z(x,t;j) = \partial_t \widehat{B}(x,t;j) - \partial_t \widetilde{B}(x,t;j) = -\nabla \times \left(\widehat{E}(x,t;j) - \widetilde{E}(x,t;j) \right) \\
&= -\nabla \times \left(\gamma_1(x) \left(\widehat{D}(x,t;j) - \widetilde{D}(x,t;j) \right) + \gamma_2(x) \left(\widehat{B}(x,t;j) - \widetilde{B}(x,t;j) \right) \right. \\
&\quad \left. - f_1(x) \widetilde{D}(x,t;j) - f_2(x) \widetilde{B}(x,t;j) \right).
\end{aligned} \tag{3.18}
$$

Differentiating (3.17) and (3.18) with respect to t and noting (3.7) and (3.13), we obtain (3.9) and (3.10). Moreover, differentiating (3.9) with respect to t and using (3.9), (3.10) and the equality :

$$
\nabla \times (aA) = a \nabla \times A + (\nabla a) \times A \tag{3.19}
$$

for a scalar function a and a vector function A of x, we have

$$
\begin{aligned}
&\partial_t^2 Y(x,t;j) = \nabla \times \left(\gamma_2(x) \left(\partial_t Y(x,t;j) \right) + \gamma_3(x) \left(\partial_t Z(x,t;j) \right) \right) - \partial_t \Psi_2(x,t;j) \\
&= \gamma_2(x) \left(\nabla \times \left(\partial_t Y(x,t;j) \right) \right) + \gamma_3(x) \left(\nabla \times \left(\partial_t Z(x,t;j) \right) \right) + \left(\nabla \gamma_2(x) \right) \times \left(\partial_t Y(x,t;j) \right) \\
&\quad + \left(\nabla \gamma_3(x) \right) \times \left(\partial_t Z(x,t;j) \right) - \partial_t \Psi_2(x,t;j) \\
&= \gamma_2(x) \left(\nabla \times \left(\nabla \times \left(\gamma_2(x) Y(x,t;j) + \gamma_3(x) Z(x,t;j) \right) \right) \right) - \gamma_2(x) \left(\nabla \times \Psi_2(x,t;j) \right) \\
&\quad + \gamma_3(x) \left(\nabla \times \left(-\nabla \times \left(\gamma_1(x) Y(x,t;j) + \gamma_2(x) Z(x,t;j) \right) \right) \right) + \gamma_3(x) \left(\nabla \times \Psi_1(x,t;j) \right) \\
&\quad + \left(\nabla \gamma_2(x) \right) \times \left(\partial_t Y(x,t;j) \right) + \left(\nabla \gamma_3(x) \right) \times \left(\partial_t Z(x,t;j) \right) - \partial_t \Psi_2(x,t;j).
\end{aligned}
$$

By using (3.19) again, we have

$$
\begin{aligned}
\partial_t^2 Y(x,t;j) = &-\left(\gamma_1(x)\gamma_3(x)-\gamma_2^2(x)\right)\nabla\times(\nabla\times Y(x,t;j)) \\
&+\{\gamma_2(x)\left[(\nabla\gamma_2(x))\times(\nabla\times Y(x,t;j))+(\nabla\gamma_3(x))\times(\nabla\times Z(x,t;j))\right. \\
&\quad\left.+\nabla\times((\nabla\gamma_2(x))\times Y(x,t;j)+(\nabla\gamma_3(x))\times Z(x,t;j))\right] \\
&-\gamma_3(x)\left[(\nabla\gamma_1(x))\times(\nabla\times Y(x,t;j))+(\nabla\gamma_2(x))\times(\nabla\times Z(x,t;j))\right. \\
&\quad\left.+\nabla\times((\nabla\gamma_1(x))\times Y(x,t;j)+(\nabla\gamma_2(x))\times Z(x,t;j))\right] \\
&\quad+\partial_t\left[(\nabla\gamma_2(x))\times Y(x,t;j)+(\nabla\gamma_3(x))\times Z(x,t;j)\right]\} \\
&-\gamma_2(x)\left(\nabla\times\Psi_2(x,t;j)\right)+\gamma_3(x)\left(\nabla\times\Psi_1(x,t;j)\right)-\partial_t\Psi_2(x,t;j).
\end{aligned} \tag{3.20}
$$

Therefore, multiplying (3.20) by $\xi(x)$ and using (3.4)-(3.5), (3.8), (3.14)-(3.15) and the equality: $\nabla\times(\nabla\times Y)=\nabla(\nabla\cdot Y)-\Delta Y$, we obtain (3.11). Similarly, we can obtain (3.12).

By (1.4) and (2.1), we have

$$
\begin{cases}
\varphi(x,0)\geq 1, & x\in\overline{\Omega}, \\
0<\varphi(x,-T)=\varphi(x,T)<1, & x\in\overline{\Omega}.
\end{cases}
$$

Therefore, for any small $\eta>0$, we can choose a sufficiently small $\delta=\delta(\eta)>0$, such that

$$
\begin{cases}
\varphi(x,t)\geq 1-\eta, & x\in\overline{\Omega},\ t\in[-\delta,\delta], \\
\varphi(x,t)\leq 1-2\eta, & x\in\overline{\Omega},\ t\in[-T,-T+2\delta]\cup[T-2\delta,T].
\end{cases} \tag{3.21}
$$

In order to apply the Carleman estimate, we introduce two cut-off functions χ_1 and χ_2 satisfying $0\leq\chi_1,\chi_2\leq 1$, $\chi_1\in C^\infty(\mathbb{R})$, $\chi_2\in C_0^\infty(\Omega)$, $\chi_2=1$ on $\Omega\backslash\omega$, and

$$
\chi_1(t)=\begin{cases}
0, & t\in[-T,-T+\delta]\cup[T-\delta,T], \\
1, & t\in[-T+2\delta,T-2\delta].
\end{cases}
$$

Furthermore, we let $\chi(x,t)=\chi_1(t)\chi_2(x)$.

For $j=1,2$, we set

$$
\begin{cases}
Y_1(x,t;j)=Y(x,t;j)\mathrm{e}^{s\varphi(x,t)}\chi(x,t)\in\left(C^2\left(\Omega\times(-T,T)\right)\right)^3, \\
Z_1(x,t;j)=Z(x,t;j)\mathrm{e}^{s\varphi(x,t)}\chi(x,t)\in\left(C^2\left(\Omega\times(-T,T)\right)\right)^3.
\end{cases} \tag{3.22}
$$

By (3.9)-(3.10), the vector functions $Y_1(x,t;j)$ and $Z_1(x,t;j)$ satisfy the equations

$$\begin{aligned}
&\partial_t Y_1(x,t;j) - \nabla\times(\gamma_2(x)Y_1(x,t;j)+\gamma_3(x)Z_1(x,t;j))\\
&= -\mathrm{e}^{s\varphi(x,t)}\chi(x,t)\Psi_2(x,t;j) + s\left(\partial_t\varphi(x,t)\right)Y_1(x,t;j)\\
&-s\left((\nabla\varphi(x,t))\times(\gamma_2(x)Y_1(x,t;j)+\gamma_3(x)Z_1(x,t;j))\right)\\
&+\mathrm{e}^{s\varphi(x,t)}\left((\partial_t\chi(x,t))\,Y(x,t;j) - (\nabla\chi(x,t))\times(\gamma_2(x)Y(x,t;j)+\gamma_3(x)Z(x,t;j))\right),
\end{aligned}\tag{3.23}$$

$$\begin{aligned}
&\partial_t Z_1(x,t;j) + \nabla\times(\gamma_1(x)Y_1(x,t;j)+\gamma_2(x)Z_1(x,t;j))\\
&= \mathrm{e}^{s\varphi(x,t)}\chi(x,t)\Psi_2(x,t;j) + s\left(\partial_t\varphi(x,t)\right)Z_1(x,t;j)\\
&+s\left((\nabla\varphi(x,t))\times(\gamma_1(x)Y_1(x,t;j)+\gamma_2(x)Z_1(x,t;j))\right)\\
&+\mathrm{e}^{s\varphi(x,t)}\left((\partial_t\chi(x,t))\,Z(x,t;j) + (\nabla\chi(x,t))\times(\gamma_1(x)Y(x,t;j)+\gamma_2(x)Z(x,t;j))\right),
\end{aligned}\tag{3.24}$$

where $x\in\Omega$, $-T<t<T$. In fact, we have

$$\begin{aligned}
&\partial_t Y_1(x,t;j) = \left(\partial_t Y(x,t;j)\right)\mathrm{e}^{s\varphi(x,t)}\chi(x,t)\\
&\quad + s\left(\partial_t\varphi(x,t)\right)Y(x,t;j)\mathrm{e}^{s\varphi(x,t)}\chi(x,t) + \mathrm{e}^{s\varphi(x,t)}\left(\partial_t\chi(x,t)\right)Y(x,t;j)\\
&= \left(\partial_t Y(x,t;j)\right)\mathrm{e}^{s\varphi(x,t)}\chi(x,t) + s\left(\partial_t\varphi(x,t)\right)Y_1(x,t;j) + \mathrm{e}^{s\varphi(x,t)}\left(\partial_t\chi(x,t)\right)Y(x,t;j).
\end{aligned}$$

Moreover, we have

$$\begin{aligned}
&\nabla\times(\gamma_2(x)Y_1(x,t;j)+\gamma_3(x)Z_1(x,t;j))\\
&= \nabla\times\left(\mathrm{e}^{s\varphi(x,t)}\chi(x,t)\left(\gamma_2(x)Y(x,t;j)+\gamma_3(x)Z(x,t;j)\right)\right)\\
&= \left(\nabla\left(\mathrm{e}^{s\varphi(x,t)}\chi(x,t)\right)\right)\times(\gamma_2(x)Y(x,t;j)+\gamma_3(x)Z(x,t;j))\\
&\quad +\mathrm{e}^{s\varphi(x,t)}\chi(x,t)\left(\nabla\times(\gamma_2(x)Y(x,t;j)+\gamma_3(x)Z(x,t;j))\right)\\
&= s\mathrm{e}^{s\varphi(x,t)}\chi(x,t)\left((\nabla\varphi(x,t))\times(\gamma_2(x)Y(x,t;j)+\gamma_3(x)Z(x,t;j))\right)\\
&\quad +\mathrm{e}^{s\varphi(x,t)}\left((\nabla\chi(x,t))\times(\gamma_2(x)Y(x,t;j)+\gamma_3(x)Z(x,t;j))\right)\\
&\quad +\mathrm{e}^{s\varphi(x,t)}\chi(x,t)\left(\nabla\times(\gamma_2(x)Y(x,t;j)+\gamma_3(x)Z(x,t;j))\right)\\
&= \mathrm{e}^{s\varphi(x,t)}\chi(x,t)\left(\nabla\times(\gamma_2(x)Y(x,t;j)+\gamma_3(x)Z(x,t;j))\right)\\
&\quad +s\left((\nabla\varphi(x,t))\times(\gamma_2(x)Y_1(x,t;j)+\gamma_3(x)Z_1(x,t;j))\right)\\
&\quad +\mathrm{e}^{s\varphi(x,t)}\left((\nabla\chi(x,t))\times(\gamma_2(x)Y(x,t;j)+\gamma_3(x)Z(x,t;j))\right).
\end{aligned}$$

At the second equality, we have used (3.19). Therefore, we have

$$\begin{aligned}
&\partial_t Y_1(x,t;j) - \nabla\times(\gamma_2(x)Y_1(x,t;j)+\gamma_3(x)Z_1(x,t;j))\\
&= \mathrm{e}^{s\varphi(x,t)}\chi(x,t)\left\{\partial_t Y(x,t;j) - \nabla\times(\gamma_2(x)Y(x,t;j)+\gamma_3(x)Z(x,t;j))\right\}\\
&\quad + s\left(\partial_t\varphi(x,t)\right)Y_1(x,t;j)\\
&\quad - s\left((\nabla\varphi(x,t))\times(\gamma_2(x)Y_1(x,t;j)+\gamma_3(x)Z_1(x,t;j))\right)\\
&\quad + \mathrm{e}^{s\varphi(x,t)}\left((\partial_t\chi(x,t))\,Y(x,t;j) - (\nabla\chi(x,t))\times(\gamma_2(x)Y(x,t;j)+\gamma_3(x)Z(x,t;j))\right).
\end{aligned}$$

Hence, by (3.9), we obtain (3.23). Moreover, by noting (3.10), we can similarly obtain (3.24).

By (3.23) and (3.24), we can obtain

$$\begin{aligned}
&\int_{-T}^{0}\int_{\Omega}\xi(x)\left\{[\partial_t Y_1(x,t;j) - \nabla\times(\gamma_2(x)Y_1(x,t;j)+\gamma_3(x)Z_1(x,t;j))]\right.\\
&\cdot[\gamma_1(x)Y_1(x,t;j)+\gamma_2(x)Z_1(x,t;j)] + [\partial_t Z_1(x,t;j) + \nabla\times(\gamma_1(x)Y_1(x,t;j)\\
&\left.+\gamma_2(x)Z_1(x,t;j))]\cdot[\gamma_2(x)Y_1(x,t;j)+\gamma_3(x)Z_1(x,t;j)]\right\}\mathrm{d}x\mathrm{d}t\\
&= \int_{-T}^{0}\int_{\Omega}\xi(x)\left\{\left[s(\partial_t\varphi(x,t))Y_1(x,t;j)\right.\right.\\
&-s\left((\nabla\varphi(x,t))\times(\gamma_2(x)Y_1(x,t;j)+\gamma_3(x)Z_1(x,t;j))\right)\\
&+\mathrm{e}^{s\varphi(x,t)}\left((\partial_t\chi(x,t))\,Y(x,t;j) - (\nabla\chi(x,t))\times(\gamma_2(x)Y(x,t;j)+\gamma_3(x)Z(x,t;j))\right)\\
&\left.-\mathrm{e}^{s\varphi(x,t)}\chi(x,t)\Psi_2(x,t;j)\right]\cdot[\gamma_1(x)Y_1(x,t;j)+\gamma_2(x)Z_1(x,t;j)]\\
&+\left[s\left((\nabla\varphi(x,t))\times(\gamma_1(x)Y_1(x,t;j)+\gamma_2(x)Z_1(x,t;j))\right) + s(\partial_t\varphi(x,t))Z_1(x,t;j)\right.\\
&+\mathrm{e}^{s\varphi(x,t)}\left((\partial_t\chi(x,t))\,Z(x,t;j) + (\nabla\chi(x,t))\times(\gamma_1(x)Y(x,t;j)+\gamma_2(x)Z(x,t;j))\right)\\
&\left.\left.+\mathrm{e}^{s\varphi(x,t)}\chi(x,t)\Psi_1(x,t;j)\right]\cdot[\gamma_2(x)Y_1(x,t;j)+\gamma_3(x)Z_1(x,t;j)]\right\}\mathrm{d}x\mathrm{d}t.
\end{aligned}\tag{3.25}$$

We denote the left- and the right-hand sides of (3.25), respectively, by $I_1(j)$ and $I_2(j)$. Using (3.4)-(3.5), (3.14) and, for vector functions A_1 and A_2, $(\nabla\times A_1)\cdot A_2 - (\nabla\times A_2)\cdot A_1 =$

$\nabla\cdot(A_1\times A_2)$, $A_1\times A_1=0$, $A_1\times A_2=-A_2\times A_1$, we have

$$\begin{aligned}
&\xi(x)\left\{[\partial_t Y_1(x,t;j)-\nabla\times(\gamma_2(x)Y_1(x,t;j)+\gamma_3(x)Z_1(x,t;j))]\right.\\
&\cdot[\gamma_1(x)Y_1(x,t;j)+\gamma_2(x)Z_1(x,t;j)]+[\partial_t Z_1(x,t;j)+\nabla\times(\gamma_1(x)Y_1(x,t;j)\\
&\left.+\gamma_2(x)Z_1(x,t;j))]\cdot[\gamma_2(x)Y_1(x,t;j)+\gamma_3(x)Z_1(x,t;j)]\right\}\\
&=\tfrac{1}{2}\partial_t\left\{\mu(x)|Y_1(x,t;j)|^2-2\zeta(x)\left(Y_1(x,t;j)\cdot Z_1(x,t;j)\right)+\epsilon(x)|Z_1(x,t;j)|^2\right\}\\
&+\xi(x)\left\{\nabla\cdot\left[\tfrac{1}{\xi(x)}\left(Y_1(x,t;j)\times Z_1(x,t;j)\right)\right]\right\}.
\end{aligned}$$

Moreover, by (3.22) and the definition of $\chi(x,t)$, we have

$$Y_1(x,-T;j)=Z_1(x,-T;j)=0,\quad x\in\Omega,$$

$$Y_1(x,t;j)=Z_1(x,t;j)=0,\quad x\in\partial\Omega,\quad -T<t<T.$$

Hence, by noting $(\epsilon,\zeta,\mu)\in\mathcal{U}_{\beta,M,\theta_0,\theta_1,\epsilon_0,\zeta_0,\mu_0}$, integrating $I_1(j)$ by parts yields

$$\begin{aligned}
I_1(j)&=\frac{1}{2}\int_\Omega\left(\mu(x)|Y_1(x,0;j)|^2-2\zeta(x)\left(Y_1(x,0;j)\cdot Z_1(x,0;j)\right)\right.\\
&\left.+\epsilon(x)|Z_1(x,0;j)|^2\right)\mathrm{d}x-\int_{-T}^0\int_\Omega\frac{1}{\xi(x)}\left\{(Y_1(x,t;j)\times Z_1(x,t;j))\cdot(\nabla\xi(x))\right\}\mathrm{d}x\mathrm{d}t\\
&\ge\frac{1}{2}\int_\Omega\left(\mu(x)|Y_1(x,0;j)|^2-2\zeta(x)\left(Y_1(x,0;j)\cdot Z_1(x,0;j)\right)+\epsilon(x)|Z_1(x,0;j)|^2\right)\mathrm{d}x\\
&-C_1\int_{-T}^T\int_\Omega\frac{1}{\xi(x)}\left|(Y_1(x,t;j)\times Z_1(x,t;j))\cdot(\nabla\xi(x))\right|\mathrm{d}x\mathrm{d}t\\
&\ge\frac{1}{2}\int_\Omega\left(\mu(x)|Y_1(x,0;j)|^2-2\zeta(x)\left(Y_1(x,0;j)\cdot Z_1(x,0;j)\right)+\epsilon(x)|Z_1(x,0;j)|^2\right)\mathrm{d}x\\
&-C_2\int_{-T}^T\int_\Omega\left(|Y_1(x,t;j)|^2+|Z_1(x,t;j)|^2\right)\mathrm{d}x\mathrm{d}t.
\end{aligned}\tag{3.26}$$

Here and henceforth $C_k>0$ $(k=1,2,\dots)$ denotes generic constants depending on s_0, σ, λ, Λ, β, M, θ_0, θ_1, θ_2, ϵ_0, ζ_0, μ_0, Ω, T, ω, χ, η, δ, d_0^j, b_0^j and $\left\|\widehat{D}(\cdot,\cdot;j)\right\|_{(H^2(\Omega\times(-T,T)))^3}$, $\left\|\widehat{B}(\cdot,\cdot;j)\right\|_{(H^2(\Omega\times(-T,T)))^3}$, $\left\|\widetilde{D}(\cdot,\cdot;j)\right\|_{(H^2(\Omega\times(-T,T)))^3}$, $\left\|\widetilde{B}(\cdot,\cdot;j)\right\|_{(H^2(\Omega\times(-T,T)))^3}$, but independent of $s>s_0$. Furthermore, by noting (2.1), (3.4), $(\epsilon,\zeta,\mu)\in\mathcal{U}_{\beta,M,\theta_0,\theta_1,\epsilon_0,\zeta_0,\mu_0}$ and using Cauchy-Bunyakovskii inequality, we have

$$\begin{aligned}
I_2(j)\le C_3&\left(\int_{-T}^0\int_\Omega \mathrm{e}^{2s\varphi(x,t)}|\chi(x,t)|^2\left(\sum_{k=1}^2|\Psi_k(x,t;j)|^2\right)\mathrm{d}x\mathrm{d}t\right.\\
&\left.+\int_{-T}^0\int_\Omega s\left(|Y_1(x,t;j)|^2+|Z_1(x,t;j)|^2\right)\mathrm{d}x\mathrm{d}t+I_3(j)\right)
\end{aligned}\tag{3.27}$$

for all large $s > 0$, where

$$I_3(j) = \int_{-T}^{T}\int_{\Omega} \left(|\partial_t\chi(x,t)|^2 + |\partial_t^2\chi(x,t)|^2 + |\nabla\chi(x,t)|^2 + |\Delta\chi(x,t)|^2\right) \cdot \Bigg(|Y(x,t;j)|^2 + |Z(x,t;j)|^2 + \sum_{k=1}^{3}\left(|\partial_k Y(x,t;j)|^2 + |\partial_k Z(x,t;j)|^2\right) + |\partial_t Y(x,t;j)|^2 + |\partial_t Z(x,t;j)|^2\Bigg) \mathrm{e}^{2s\varphi(x,t)}\mathrm{d}x\mathrm{d}t \tag{3.28}$$

Therefore, it follows from $(\epsilon,\zeta,\mu)\in\mathcal{U}_{\beta,M,\theta_0,\theta_1,\epsilon_0,\zeta_0,\mu_0}$, (3.22), (3.25)-(3.27) and the definition of $\chi(x,t)$ that

$$\begin{aligned}
&\int_{\Omega}\left(|Y_1(x,0;j)|^2 + |Z_1(x,0;j)|^2\right)\mathrm{d}x \\
\le\ & C_4\int_{\Omega}\frac{1}{2}\left(\mu(x)|Y_1(x,0;j)|^2 - 2\zeta(x)\left(Y_1(x,0;j)\cdot Z_1(x,0;j)\right)\right. \\
&\quad \left. +\epsilon(x)|Z_1(x,0;j)|^2\right)\mathrm{d}x \\
\le\ & C_5\left(\int_{-T}^{T}\int_{\Omega}\mathrm{e}^{2s\varphi(x,t)}|\chi(x,t)|^2\left(\sum_{k=1}^{2}|\Psi_k(x,t;j)|^2\right)\mathrm{d}x\mathrm{d}t\right. \\
&\quad \left. +\int_{-T}^{T}\int_{\Omega} s\left(|Y_1(x,t;j)|^2 + |Z_1(x,t;j)|^2\right)\mathrm{d}x\mathrm{d}t + I_3(j)\right) \\
\le\ & C_6\left(\int_{-T}^{T}\int_{\Omega}\mathrm{e}^{2s\varphi(x,t)}\left(\sum_{k=1}^{2}|\Psi_k(x,t;j)|^2\right)\mathrm{d}x\mathrm{d}t + I_3(j)\right. \\
&\quad \left. +\int_{-T}^{T}\int_{\Omega} s\mathrm{e}^{2s\varphi(x,t)}\left(|\chi(x,t)Y(x,t;j)|^2 + |\chi(x,t)Z(x,t;j)|^2\right)\mathrm{d}x\mathrm{d}t\right)
\end{aligned} \tag{3.29}$$

for all large $s > 0$.

Next, we shall first estimate the last term of (3.29) by applying proposition 2.1. We set $U(x,t;j) = \chi(x,t)Y(x,t;j)$ and $V(x,t;j) = \chi(x,t)Z(x,t;j)$ for $x\in\Omega$, $-T < t < T$ and $j = 1,2$. Then $U(x,t;j)$ satisfies

$$\begin{aligned}
&\xi(x)\left(\partial_t^2 U(x,t;j)\right) - \Delta U(x,t;j) = \Phi_1(U(x,t;j), V(x,t;j)) \\
&+\chi(x,t)\left[\zeta(x)\left(\nabla\times\Psi_2(x,t;j)\right) + \epsilon(x)\left(\nabla\times\Psi_1(x,t;j)\right) - \xi(x)\left(\partial_t\Psi_2(x,t;j)\right)\right] \\
&+\Phi_4(Y(x,t;j), Z(x,t;j))
\end{aligned} \tag{3.30}$$

for $x \in \Omega$, $-T < t < T$, where

$$\Phi_4(Y(x,t;j), Z(x,t;j)) = \xi(x)\left(2\left(\partial_t \chi(x,t)\right)\left(\partial_t Y(x,t;j)\right) + \left(\partial_t^2 \chi(x,t)\right) Y(x,t;j)\right)$$
$$-2\left(\sum_{k=1}^{3}\left(\partial_k \chi(x,t)\right)\left(\partial_k Y(x,t;j)\right)\right) - \left(\Delta \chi(x,t)\right) Y(x,t;j) + \Phi_3(Y(x,t;j), Z(x,t;j)) \tag{3.31}$$

and $\Phi_3(Y(x,t;j), Z(x,t;j))$ will be defined by (3.34). In fact, by directly calculating, we can see that

$$\begin{aligned} &\xi(x)\left(\partial_t^2 U(x,t;j)\right) - \Delta U(x,t;j) \\ &= \chi(x,t)\left(\xi(x)\left(\partial_t^2 Y(x,t;j)\right) - \Delta Y(x,t;j)\right) \\ &+\xi(x)\left(2\left(\partial_t \chi(x,t)\right)\left(\partial_t Y(x,t;j)\right) + \left(\partial_t^2 \chi(x,t)\right) Y(x,t;j)\right) \\ &-2\left(\textstyle\sum_{k=1}^{3}\left(\partial_k \chi(x,t)\right)\left(\partial_k Y(x,t;j)\right)\right) - \left(\Delta \chi(x,t)\right) Y(x,t;j). \end{aligned} \tag{3.32}$$

Moreover, by (3.15), we have

$$\chi(x,t)\Phi_1(Y(x,t;j), Z(x,t;j)) = \Phi_1(U(x,t;j), V(x,t;j)) + \Phi_3(Y(x,t;j), Z(x,t;j)) \tag{3.33}$$

where

$$\begin{aligned} &\Phi_3(Y(x,t;j), Z(x,t;j)) = \\ &-\xi(x)\left(\partial_t \chi(x,t)\right)\left((\nabla\gamma_2(x)) \times Y(x,t;j) + (\nabla\gamma_3(x)) \times Z(x,t;j)\right) \\ &+\zeta(x)\left[(\nabla\chi(x,t)) \times ((\nabla\gamma_2(x)) \times Y(x,t;j) + (\nabla\gamma_3(x)) \times Z(x,t;j))\right. \\ &\left.+(\nabla\gamma_2(x)) \times ((\nabla\chi(x,t)) \times Y(x,t;j)) + (\nabla\gamma_3(x)) \times ((\nabla\chi(x,t)) \times Z(x,t;j))\right] \\ &+\epsilon(x)\left[(\nabla\chi(x,t)) \times ((\nabla\gamma_1(x)) \times Y(x,t;j) + (\nabla\gamma_2(x)) \times Z(x,t;j))\right. \\ &\left.+(\nabla\gamma_1(x)) \times ((\nabla\chi(x,t)) \times Y(x,t;j)) + (\nabla\gamma_2(x)) \times ((\nabla\chi(x,t)) \times Z(x,t;j))\right]. \end{aligned} \tag{3.34}$$

Therefore, by (3.11) and (3.32)-(3.33), we obtain (3.30). Hence, by (3.30) and the definition of $\chi(x,t)$ and $U(x,t;j)$, we can apply proposition 2.1 to $U(\cdot,\cdot;j)$. As a result, by noting $(\epsilon,\zeta,\mu)\in\mathcal{U}_{\beta,M,\theta_0,\theta_1,\epsilon_0,\zeta_0,\mu_0}$, (3.15), (3.28), (3.31), (3.34) and the definition of $\chi(x,t)$, we can obtain

$$
\begin{aligned}
&\int_{-T}^{T}\int_{\Omega} s|U(x,t;j)|^2 \mathrm{e}^{2s\varphi(x,t)}\mathrm{d}x\mathrm{d}t\\
&\le C_7\left(\int_{-T}^{T}\int_{\Omega}\left(\sum_{k=1}^{2}|\Psi_k(x,t;j)|^2\right)\mathrm{e}^{2s\varphi(x,t)}\mathrm{d}x\mathrm{d}t\right.\\
&+\int_{-T}^{T}\int_{\Omega}\left(|U(x,t;j)|^2+|V(x,t;j)|^2\right)\mathrm{e}^{2s\varphi(x,t)}\mathrm{d}x\mathrm{d}t\\
&\left.+\int_{-T}^{T}\int_{\Omega}|\Phi_4(Y(x,t;j),Z(x,t;j))|^2\mathrm{e}^{2s\varphi(x,t)}\mathrm{d}x\mathrm{d}t\right)\\
&\le C_8\left(\int_{-T}^{T}\int_{\Omega}\left(\sum_{k=1}^{2}|\Psi_k(x,t;j)|^2\right)\mathrm{e}^{2s\varphi(x,t)}\mathrm{d}x\mathrm{d}t\right.\\
&\left.+\int_{-T}^{T}\int_{\Omega}\left(|U(x,t;j)|^2+|V(x,t;j)|^2\right)\mathrm{e}^{2s\varphi(x,t)}\mathrm{d}x\mathrm{d}t+I_3(j)\right)
\end{aligned}
\tag{3.35}
$$

for all large $s>0$. By noting (3.12), (3.16) and using proposition 2.1, we can similarly obtain that

$$
\begin{aligned}
&\int_{-T}^{T}\int_{\Omega} s|V(x,t;j)|^2 \mathrm{e}^{2s\varphi(x,t)}\mathrm{d}x\mathrm{d}t\\
&\le C_9\left(\int_{-T}^{T}\int_{\Omega}\left(\sum_{k=1}^{2}|\Psi_k(x,t;j)|^2\right)\mathrm{e}^{2s\varphi(x,t)}\mathrm{d}x\mathrm{d}t\right.\\
&\left.+\int_{-T}^{T}\int_{\Omega}\left(|U(x,t;j)|^2+|V(x,t;j)|^2\right)\mathrm{e}^{2s\varphi(x,t)}\mathrm{d}x\mathrm{d}t+I_3(j)\right)
\end{aligned}
\tag{3.36}
$$

for all large $s>0$. Then, by (3.35) and (3.36), we can see that

$$
\begin{aligned}
&\int_{-T}^{T}\int_{\Omega} s\left(|U(x,t;j)|^2+|V(x,t;j)|^2\right)\mathrm{e}^{2s\varphi(x,t)}\mathrm{d}x\mathrm{d}t\\
&\le C_{10}\left(\int_{-T}^{T}\int_{\Omega}\left(\sum_{k=1}^{2}|\Psi_k(x,t;j)|^2\right)\mathrm{e}^{2s\varphi(x,t)}\mathrm{d}x\mathrm{d}t+I_3(j)\right)
\end{aligned}
\tag{3.37}
$$

for all sufficiently large $s>0$.

In addition, we shall estimate $I_3(j)$. By (3.7), (3.21), (3.28) and noting the definition of $\chi(x,t)$, we have

$$
\begin{aligned}
I_3(j) &\le C_{11}\left(\int_{-T}^{T}\int_\Omega \left(|\partial_t\chi(x,t)|^2+|\partial_t^2\chi(x,t)|^2\right)\left(|Y(x,t;j)|^2\right.\right.\\
&\quad +|Z(x,t;j)|^2+\sum_{k=1}^{3}\left(|\partial_k Y(x,t;j)|^2+|\partial_k Z(x,t;j)|^2\right)\\
&\quad \left.\left.+|\partial_t Y(x,t;j)|^2+|\partial_t Z(x,t;j)|^2\right)\mathrm{e}^{2s\varphi(x,t)}\mathrm{d}x\mathrm{d}t+\mathrm{e}^{2s\Gamma}\Theta\right)\\
&\le c_{12}\left(\left(\int_{-T+\delta}^{-T+2\delta}+\int_{T-2\delta}^{T-\delta}\right)\int_\Omega \left(|\partial_t\chi(x,t)|^2+|\partial_t^2\chi(x,t)|^2\right)\left(|Y(x,t;j)|^2\right.\right.\\
&\quad +|Z(x,t;j)|^2+\sum_{k=1}^{3}\left(|\partial_k Y(x,t;j)|^2+|\partial_k Z(x,t;j)|^2\right)\\
&\quad \left.\left.+|\partial_t Y(x,t;j)|^2+|\partial_t Z(x,t;j)|^2\right)\mathrm{e}^{2s\varphi(x,t)}\mathrm{d}x\mathrm{d}t+\mathrm{e}^{2s\Gamma}\Theta\right)\\
&\le c_{13}\left(\mathrm{e}^{2s(1-2\eta)}+\mathrm{e}^{2s\Gamma}\Theta\right)
\end{aligned}
\tag{3.38}
$$

for all sufficiently large $s>0$, where $\Gamma=\sup_{(x,t)\in\Omega\times(-T,T)}\varphi(x,t)$ and

$$
\Theta=\sum_{j=1}^{2}\left(\|Y(\cdot,\cdot;j)\|^2_{(H^1(\omega\times(-T,T)))^3}+\|Z(\cdot,\cdot;j)\|^2_{(H^1(\omega\times(-T,T)))^3}\right). \tag{3.39}
$$

Hence, by $(\epsilon,\zeta,\mu)\in\mathcal{U}_{\beta,M,\theta_0,\theta_1,\epsilon_0,\zeta_0,\mu_0}$, the definition of $\chi(x,t)$, $U(x,t;j)$ and $V(x,t;j)$, (3.22), (3.29), (3.37)-(3.39), we see that

$$
\begin{aligned}
&\int_\Omega \left(|Y(x,0;j)|^2+|Z(x,0;j)|^2\right)\mathrm{e}^{2s\varphi(x,0)}\mathrm{d}x\\
&\le C_{14}\left(\int_\Omega |\chi_2(x)|^2\left(|Y(x,0;j)|^2+|Z(x,0;j)|^2\right)\mathrm{e}^{2s\varphi(x,0)}\mathrm{d}x+\mathrm{e}^{2s\Gamma}\Theta\right)\\
&= C_{14}\left(\int_\Omega \left(|Y_1(x,0;j)|^2+|Z_1(x,0;j)|^2\right)\mathrm{d}x+\mathrm{e}^{2s\Gamma}\Theta\right)\\
&\le C_{15}\left(\int_{-T}^{T}\int_\Omega\left(\sum_{k=1}^{2}|\Psi_k(x,t;j)|^2\right)\mathrm{e}^{2s\varphi(x,t)}\mathrm{d}x\mathrm{d}t+\mathrm{e}^{2s(1-2\eta)}+\mathrm{e}^{2s\Gamma}\Theta\right)\\
&\le C_{16}\left(\int_{-T}^{T}\int_\Omega \mathrm{e}^{2s\varphi(x,t)}\sum_{k=1}^{3}\left(|f_k(x)|^2+|\nabla f_k(x)|^2\right)\mathrm{d}x\mathrm{d}t+\mathrm{e}^{2s(1-2\eta)}+\mathrm{e}^{2s\Gamma}\Theta\right)
\end{aligned}
\tag{3.40}
$$

for all sufficiently large $s>0$ and $j=1,2$. At the last inequality, We have used (3.2), (3.13) and (3.19).

On the other hand, by (1.1) and (3.1), for $j=1,2$, we have

$$
\widehat{D}(x,0;j)=\widetilde{D}(x,0;j)=d^j(x),\ \ \widehat{B}(x,0;j)=\widetilde{B}(x,0;j)=b^j(x),\quad x\in\Omega.
$$

Therefore, by (3.17) and (3.18), we have

$$
\begin{aligned}
& Y(x,0;j) = -\nabla\times\left(f_2(x)d^j(x)+f_3(x)b^j(x)\right)\\
& = -f_2(x)\left(\nabla\times d^j(x)\right) - f_3(x)\left(\nabla\times b^j(x)\right) - \left(\partial_1 f_2(x)\right)\left(e_1\times d^j(x)\right)\\
& -\left(\partial_1 f_3(x)\right)\left(e_1\times b^j(x)\right) - \left(\partial_2 f_2(x)\right)\left(e_2\times d^j(x)\right) - \left(\partial_2 f_3(x)\right)\left(e_2\times b^j(x)\right)\\
& -\left(\partial_3 f_2(x)\right)\left(e_3\times d^j(x)\right) - \left(\partial_3 f_3(x)\right)\left(e_3\times b^j(x)\right),\ x\in\Omega,
\end{aligned}
$$

$$
\begin{aligned}
& Z(x,0;j) = \nabla\times\left(f_1(x)d^j(x)+f_2(x)b^j(x)\right)\\
& = f_1(x)\left(\nabla\times d^j(x)\right) + f_2(x)\left(\nabla\times b^j(x)\right) + \left(\partial_1 f_1(x)\right)\left(e_1\times d^j(x)\right)\\
& +\left(\partial_1 f_2(x)\right)\left(e_1\times b^j(x)\right) + \left(\partial_2 f_1(x)\right)\left(e_2\times d^j(x)\right) + \left(\partial_2 f_2(x)\right)\left(e_2\times b^j(x)\right)\\
& +\left(\partial_3 f_1(x)\right)\left(e_3\times d^j(x)\right) + \left(\partial_3 f_2(x)\right)\left(e_3\times b^j(x)\right),\ x\in\Omega.
\end{aligned}
$$

Then, we have

$$
\mathbf{G}F(x) = \begin{pmatrix} -Y(x,0;1)\\ Z(x,0;1)\\ -Y(x,0;2)\\ Z(x,0;2)\end{pmatrix} - \begin{pmatrix} 0 & \nabla\times d^1(x) & \nabla\times b^1(x)\\ \nabla\times d^1(x) & \nabla\times b^1(x) & 0\\ 0 & \nabla\times d^2(x) & \nabla\times b^2(x)\\ \nabla\times d^2(x) & \nabla\times b^2(x) & 0\end{pmatrix}\begin{pmatrix} f_1(x)\\ f_2(x)\\ f_3(x)\end{pmatrix} \tag{3.41}
$$

where $F(x) = \left(\partial_1 f_1, \partial_1 f_2, \partial_1 f_3, \partial_2 f_1, \partial_2 f_2, \partial_2 f_3, \partial_3 f_1, \partial_3 f_2, \partial_3 f_3\right)^T(x)$ and $x\in\Omega$. By (1.5) and (3.41), we see that

$$
\sum_{k=1}^{3}\left(|f_k(x)|^2+|\nabla f_k(x)|^2\right) \le C_{17}\left(\sum_{j=1}^{2}\left(|Y(x,0;j)|^2+|Z(x,0;j)|^2\right)+\sum_{k=1}^{3}|f_k(x)|^2\right). \tag{3.42}
$$

Therefore, by (3.40) and (3.42), we have

$$
\begin{aligned}
& \int_\Omega e^{2s\varphi(x,0)}\sum_{k=1}^{3}\left(|f_k(x)|^2+|\nabla f_k(x)|^2\right)\mathrm{d}x\\
& \le C_{18}\int_\Omega e^{2s\varphi(x,0)}\left(\sum_{j=1}^{2}\left(|Y(x,0;j)|^2+|Z(x,0;j)|^2\right)+\sum_{k=1}^{3}|f_k(x)|^2\right)\mathrm{d}x.\\
& \le C_{19}\left(\int_\Omega e^{2s\varphi(x,0)}\sum_{k=1}^{3}|f_k(x)|^2\mathrm{d}x\right.\\
& \quad \left. + \int_{-T}^{T}\int_\Omega e^{2s\varphi(x,t)}\sum_{k=1}^{3}\left(|f_k(x)|^2+|\nabla f_k(x)|^2\right)\mathrm{d}x\mathrm{d}t + e^{2s(1-2\eta)} + e^{2s\Gamma}\Theta\right)
\end{aligned} \tag{3.43}
$$

for all sufficiently large $s > 0$. Furthermore, by noting (ϵ, ζ, μ), $(\widetilde{\epsilon}, \widetilde{\zeta}, \widetilde{\mu}) \in \mathcal{U}_{\beta, M, \theta_0, \theta_1, \epsilon_0, \zeta_0, \mu_0}$, we can apply proposition 2.2 to $f_k(x)$ $(k = 1, 2, 3)$. As a result, we obtain

$$\int_\Omega \mathrm{e}^{2s\varphi(x,0)} \sum_{k=1}^{3} |f_k(x)|^2 \mathrm{d}x \le \frac{C_{20}}{s} \int_\Omega \sum_{k=1}^{3} |\nabla f_k|^2 \, \mathrm{e}^{2s\varphi(x,0)} \mathrm{d}x \tag{3.44}$$

for all sufficiently large $s > 0$. Then it follows from (3.43) and (3.44) that

$$\begin{aligned}
&\int_\Omega \mathrm{e}^{2s\varphi(x,0)} \sum_{k=1}^{3} \left(|f_k(x)|^2 + |\nabla f_k(x)|^2\right) \mathrm{d}x \\
&\le C_{21} \left(\int_{-T}^{T} \int_\Omega \sum_{k=1}^{3} \left(|f_k(x)|^2 + |\nabla f_k(x)|^2\right) \mathrm{e}^{2s\varphi(x,t)} \mathrm{d}x\mathrm{d}t + \mathrm{e}^{2s(1-2\eta)} + \mathrm{e}^{2s\Gamma}\Theta \right) \\
&\le C_{22} \left(\int_\Omega \mathrm{e}^{2s\varphi(x,0)} \sum_{k=1}^{3} \left(|f_k(x)|^2 + |\nabla f_k(x)|^2\right) \left(\int_{-T}^{T} \mathrm{e}^{2s(\varphi(x,t)-\varphi(x,0))} \mathrm{d}t \right) \mathrm{d}x \right. \\
&\quad \left. + \, \mathrm{e}^{2s(1-2\eta)} + \mathrm{e}^{2s\Gamma}\Theta \right)
\end{aligned} \tag{3.45}$$

for all sufficiently large $s > 0$. By (2.1), we have $\varphi(x,t) - \varphi(x,0) < 0$ when $t \ne 0$. Hence the Lebesgue theorem implies

$$\int_{-T}^{T} \mathrm{e}^{2s(\varphi(x,t)-\varphi(x,0))} \mathrm{d}t \to 0 \tag{3.46}$$

as $s \to \infty$. By (3.45) and (3.46), we can obtain that

$$\int_\Omega \mathrm{e}^{2s\varphi(x,0)} \sum_{k=1}^{3} \left(|f_k(x)|^2 + |\nabla f_k(x)|^2\right) \mathrm{d}x \le C_{23} \left(\mathrm{e}^{2s(1-2\eta)} + \mathrm{e}^{2s\Gamma}\Theta \right)$$

for all sufficiently large $s > 0$. Therefore, by noting (3.21), we have

$$\begin{aligned}
&\int_\Omega \sum_{k=1}^{3} \left(|f_k(x)|^2 + |\nabla f_k(x)|^2\right) \mathrm{d}x \\
&\le C_{24} \mathrm{e}^{-2s(1-\eta)} \int_\Omega \mathrm{e}^{2s\varphi(x,0)} \sum_{k=1}^{3} \left(|f_k(x)|^2 + |\nabla f_k(x)|^2\right) \mathrm{d}x \\
&\le C_{25} \left(\mathrm{e}^{-2s\eta} + \mathrm{e}^{2s\Gamma}\Theta \right)
\end{aligned} \tag{3.47}$$

for all sufficiently large $s > 0$. Moreover, by $(\epsilon, \zeta, \mu) \in \mathcal{U}_{\beta, M, \theta_0, \theta_1, \epsilon_0, \zeta_0, \mu_0}$, (3.4), (3.6), (3.14) and letting $\widetilde{\xi}(x) = \widetilde{\epsilon}(x)\widetilde{\mu}(x) - \widetilde{\zeta}^2(x) = \frac{1}{\widetilde{\gamma}_1(x)\widetilde{\gamma}_3(x) - \widetilde{\gamma}_2^2(x)}$ for $x \in \Omega$, we have

$$\begin{cases}
\widetilde{\epsilon}(x) - \epsilon(x) = \widetilde{\xi}(x)\widetilde{\gamma}_3(x) - \xi(x)\gamma_3(x) = \widetilde{\xi}(x) f_3(x) + \left(\widetilde{\xi}(x) - \xi(x)\right) \gamma_3(x), \\
\zeta(x) - \widetilde{\zeta}(x) = \widetilde{\xi}(x)\widetilde{\gamma}_2(x) - \xi(x)\gamma_2(x) = \widetilde{\xi}(x) f_2(x) + \left(\widetilde{\xi}(x) - \xi(x)\right) \gamma_2(x), \\
\widetilde{\mu}(x) - \mu(x) = \widetilde{\xi}(x)\widetilde{\gamma}_1(x) - \xi(x)\gamma_1(x) = \widetilde{\xi}(x) f_1(x) + \left(\widetilde{\xi}(x) - \xi(x)\right) \gamma_1(x).
\end{cases} \tag{3.48}$$

Then, by (3.5)-(3.6) and directly calculating, we have

$$\begin{aligned}\widetilde{\xi}(x)-\xi(x) &= \tfrac{1}{\widetilde{\gamma}_1(x)\widetilde{\gamma}_3(x)-\widetilde{\gamma}_2^2(x)}-\tfrac{1}{\gamma_1(x)\gamma_3(x)-\gamma_2^2(x)} \\ &= \widetilde{\xi}(x)\xi(x)\left(\left(\widetilde{\gamma}_2(x)+\gamma_2(x)\right)f_2(x)-\gamma_1(x)f_3(x)-\widetilde{\gamma}_3(x)f_1(x)\right).\end{aligned} \tag{3.49}$$

Therefore, by $(\epsilon,\zeta,\mu)\in\mathcal{U}_{\beta,M,\theta_0,\theta_1,\epsilon_0,\zeta_0,\mu_0}$, (3.14), (3.47)-(3.49), we have

$$\|\epsilon-\widetilde{\epsilon}\|^2_{L^2(\Omega)}+\left\|\zeta-\widetilde{\zeta}\right\|^2_{L^2(\Omega)}+\|\mu-\widetilde{\mu}\|^2_{L^2(\Omega)}\le c_{26}\left(\mathrm{e}^{-2s\eta}+\mathrm{e}^{2s\Gamma}\Theta\right). \tag{3.50}$$

In order to prove (1.6), we may assume that Θ is sufficiently small. So $-\frac{\ln\Theta}{2(\eta+\Gamma)}$ is sufficiently large. Therefore we can take

$$s=-\frac{\ln\Theta}{2(\eta+\Gamma)} \tag{3.51}$$

in (3.50). Then by directly calculating, we see that

$$\mathrm{e}^{-2s\eta}=\mathrm{e}^{2s\Gamma}\Theta=\Theta^{\frac{\eta}{\eta+\Gamma}}. \tag{3.52}$$

By (3.7), (3.39), (3.50) and (3.52), we obtain (1.6) with $\kappa=\frac{\eta}{\eta+\Gamma}$. The proof of theorem 1.1 is complete.

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ADDRESS:
Graduate School of Mathematical Sciences, The University of Tokyo
3–8–1 Komaba Meguro-ku, Tokyo 153-8914, JAPAN
TEL +81-3-5465-7001 FAX +81-3-5465-7012