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The abelianization of the congruence IA-automorphism group of a free group

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THE ABELIANIZATION OF THE CONGRUENCE IA-AUTOMORPHISM GROUP OF A FREE GROUP

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Dedecated to Professor Yukio Matsumoto on the occation of his 60th birthday

Abstract: Let F_n be a free group of rank n. An automorphism of F_n is called an IA-automorphism if it trivially acts on the abelianization H of F_n. We denote by IA_n the group of IA-automorphisms and call it the IA-automorphism group of F_n. For any integer $d \geq 2$, let $IA_{n,d}$ be the group of automorphisms of F_n which trivially acts on $H \otimes_{\mathbf{Z}} \mathbf{Z}/d\mathbf{Z}$. We call $IA_{n,d}$ the congruence IA-automorphism group of F_n of level d. In this paper we determine the abelianization of $IA_{n,d}$ for $n \geq 2$ and $d \geq 2$. Furthermore, for any odd prime integer p, we give some remarks on the (co)homology groups of $IA_{n,p}$ with trivial coefficients. In particullar, we show that the second cohomology group of $IA_{n,p}$ has non-trivial p-torsion elements for $n \geq 9$ and, we completely calculate the homology groups of $IA_{2,p}$ for any dimension.

Keywords: IA-automorphism group of a free group, congruence subgroup, the first Johnson homomorphism

1. Introduction

Let F_n be a free group of rank n and $\mathrm{Aut}\,F_n$ the automorphism group of the group F_n. It is well known facts that the braid group B_n of index $n \geq 3$ is embedded in $\mathrm{Aut}\,F_n$ (See [2].) and the mapping class group $\mathcal{M}_{g,1}$ of a compact oriented surface $\Sigma_{g,1}$ of genus $g \geq 2$ with one boundary component is embedded in $\mathrm{Aut}\,F_{2g}$. (See [10].) Hence it is important to study the structure and the property of $\mathrm{Aut}\,F_n$ to study those of these groups.

Our main interests are the (co)homology groups of $\mathrm{Aut}\,F_n$. There are remarkable results of the homology groups of $\mathrm{Aut}\,F_n$ with trivial coefficients. For example, Gersten [4] showed that $H_2(\mathrm{Aut}\,F_n, \mathbf{Z}) = \mathbf{Z}/2\mathbf{Z}$ for $n \geq 5$. Hatcher and Vogtmann [5] showed that $H_i(\mathrm{Aut}\,F_n, \mathbf{Q}) = 0$ for $n \geq 1$ and $1 \leq i \leq 6$, except for $H_4(\mathrm{Aut}\,F_4, \mathbf{Q}) = \mathbf{Q}$. The (co)homology groups of $\mathrm{Aut}\,F_n$ are, however, still much more unknown. In this paper we consider some nomal subgroups of $\mathrm{Aut}\,F_n$ and their (co)homology groups. In order to study the (co)homology groups of $\mathrm{Aut}\,F_n$, it is important and useful to know those of them.

Now, let H be the abelianization of F_n. The natural map $F_n \to H$ induces a homomorohism $\rho : \mathrm{Aut}\,F_n \to GL(n, \mathbf{Z})$. Clearly, this map is surjective. The kernel IA_n of the map ρ is called the IA-automorphism group of F_n. For any integer $d \geq 2$, let $GL(n,d)$ be the general linear group over $\mathbf{Z}/d\mathbf{Z}$. If we compose the map ρ with the natural reduction map $GL(n, \mathbf{Z}) \to GL(n,d)$, we obtain a homomorphism $\mathrm{Aut}\,F_n \to GL(n,d)$ whose kernel is denoted $IA_{n,d}$, the congruence IA-automorphism group of F_n of level d.

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For the group IA_n, there is a known fact that the abelianization IA_n^{ab} of IA_n, i.e., the first homology group of IA_n with integral coefficients, is a free abelian group of rank $\frac{1}{2}n^2(n-1)$. (See [6].) In this paper, our first aim is to determine the structure of abelianization $IA_{n,d}^{\mathrm{ab}}$ of $IA_{n,d}$. Let $\Gamma(n,d)$ be the kernel of the natural map $GL(n,\mathbf{Z}) \to GL(n,d)$. The group $\Gamma(n,d)$ is called the congruence subgroup of $GL(n,\mathbf{Z})$ of level d. Our main result is

Theorem 1.1. *For $n \geq 2$ and $d \geq 2$, we have*

$$IA_{n,d}^{\mathrm{ab}} \simeq \Gamma(n,d)^{\mathrm{ab}} \bigoplus (IA_n^{\mathrm{ab}} \otimes_{\mathbf{Z}} \mathbf{Z}/d\mathbf{Z}).$$

For $n \geq 3$ and any prime integer p, Lee and Szczarba [7] determined the structure of the abelianization of $\Gamma(n,p)^{\mathrm{ab}}$ as a $SL(n,p)$-module whrer $SL(n,p)$ is the special linear group over $\mathbf{Z}/p\mathbf{Z}$. In particular, they showed that $\Gamma(n,p)^{\mathrm{ab}}$ is a finite p-group. Hence, from Theorem 1.1, we see that $IA_{n,p}^{\mathrm{ab}}$ is a finite p-group.

In section 3 we give some remarks on the (co)homology groups of $IA_{n,p}$. First, for $n \geq 9$, we show that the second cohomology group $H^2(IA_{n,p},\mathbf{Z})$ has non-trivial p-torsion elements. Next, we completely calculate the homology groups of $IA_{2,p}$:

Theorem 1.2. *For any prime integer p, we have*

$$H_q(IA_{2,p},\mathbf{Z}) = \begin{cases} \mathbf{Z} & \text{if } q=0, \\ \mathbf{Z}^{\oplus \alpha(p)} \oplus (\mathbf{Z}/p\mathbf{Z})^{\oplus 2} & \text{if } q=1, \\ \mathbf{Z}^{\oplus(2\alpha(p)-2)} & \text{if } q=2, \\ 0 & \text{if } q \geq 3 \end{cases}$$

where $\alpha(p) = 1 + \frac{(p-1)p(p+1)}{12}$.

2. The abelianization of the group $IA_{n,d}$.

In this section our aim is to prove our main theorem. Before proving Theorem 1.1 we recall generators and the abelianization of IA_n. Let $x_1,\ldots,x_n$ be a basis of a free group F_n. Magnus [8] showed that IA_n is finitely generated by automorphisms

$$K_{ij} : \begin{cases} x_i & \mapsto x_j x_i {x_j}^{-1}, \\ x_t & \mapsto x_t, \quad (t \neq i) \end{cases}$$

for any distinct members i and j of the set $\{1,2,\ldots,n\}$ and

$$K_{klm} : \begin{cases} x_k & \mapsto x_k x_l x_m {x_l}^{-1} {x_m}^{-1}, \\ x_t & \mapsto x_t, \quad (t \neq k) \end{cases}$$

for any distinct members k, l and m of the set $\{1,2,\ldots,n\}$ such that $l<m$.

Let H be the abelianization of F_n and $H^* = \mathrm{Hom}_{\mathbf{Z}}(H,\mathbf{Z})$ the dual group of H. We denote by $X_1,\ldots,X_n$ the basis of H as a free abelian group induced by the free generators $x_1,\ldots,x_n$ of F_n. We also denote by $X_1^*,\ldots,X_n^*$ the dual basis of H^*. For an IA-automorphism K, let $[K]$ denote the residue class of K in the abelianization IA_n^{ab} of IA_n. Then there is a $GL(n,\mathbf{Z})$-equivariant homomorphism $\tau_n(1) : IA_n^{\mathrm{ab}} \to H^* \otimes_{\mathbf{Z}} \Lambda^2 H$, called the first Johnson homomorphism of $\mathrm{Aut}\, F_n$, which maps the generators $[K_{ij}]$ and $[K_{klm}]$ to $X_i^* \otimes X_i \wedge X_j$ and $X_k^* \otimes X_l \wedge X_m$ respectively. (For details, see [6].) Hence we see that IA_n^{ab} is a free abelian group of rank $\frac{1}{2}n^2(n-1)$ generated by the residue classes $[K_{ij}]$ and $[K_{klm}]$.

Now, we begin to prove Theorem 1.1. First, we see that since the first Johnson homomorphism $\tau_n(1)$ is $GL(n,\mathbf{Z})$-equvariant isomorphism, $\tau_n(1)$ induces a surjective homomorphism

$$\tilde{\tau}_n(1) : H_0(\Gamma(n,d), IA_n^{\mathrm{ab}}) \to (H^* \otimes_{\mathbf{Z}} \Lambda^2 H) \otimes_{\mathbf{Z}} \mathbf{Z}/d\mathbf{Z}.$$

To show $\tilde{\tau}_n(1)$ is an isomorphism, we prepare

Lemma 2.1. *For $n \geq 2$ and $d \geq 2$, we have*

$$d\,[K_{ij}] = 0 \quad \text{and} \quad d\,[K_{klm}] = 0$$

in $H_0(\Gamma(n,d), IA_n^{\mathrm{ab}})$.

Proof of Lemma 2.1. First we show $d\,[K_{ij}] = 0$. We denote by $E_{ij} \in \mathrm{Aut}\,F_n$ the Nielsen automorphism which maps x_i to $x_i x_j$ and fix x_t for $t \neq i$. Then we see that ${E_{ij}}^d \in IA_{n,d}$ and it holds

$${E_{ij}}^{-d} K_{ji} {E_{ij}}^d = {K_{ij}}^d K_{ji}$$

in IA_n. Here we note that in $\mathrm{Aut}\,F_n$, the composition of two maps E and $F \in \mathrm{Aut}\,F_n$ are defined by $(x)(E\,F) = ((x)E)F$ for any $x \in F_n$. Hence if we put $\sigma = \rho({E_{ij}}^{-d}) \in \Gamma(n,d)$ then we have

$$\sigma \cdot [K_{ji}] - [K_{ji}] = d\,[K_{ij}]$$

in IA_n^{ab}. This shows that $d\,[K_{ij}] = 0$ in $H_0(\Gamma(n,d), IA_n^{\mathrm{ab}})$.

Similarly, put $\tau = \rho({E_{kl}}^{-d}) \in \Gamma(n,d)$. Then, we have

$$\begin{aligned} &{E_{kl}}^{-d} K_{lm} {E_{kl}}^d \\ &\quad = K_{lm} K_{klm}\, K_{kl} K_{klm} {K_{kl}}^{-1}\, {K_{kl}}^2 K_{klm} {K_{kl}}^{-2} \cdots {K_{kl}}^{d-1} K_{klm} {K_{kl}}^{-(d-1)} \end{aligned}$$

in IA_n and

$$\tau \cdot [K_{lm}] - [K_{lm}] = d\,[K_{klm}]$$

in IA_n^{ab}. This shows that $d\,[K_{klm}] = 0$ in $H_0(\Gamma(n,d), IA_n^{\mathrm{ab}})$. This completes the proof of Lemma.

From this Lemma, we can define a homomorphism

$$\mu : (H^* \otimes_{\mathbf{Z}} \Lambda^2 H) \otimes_{\mathbf{Z}} \mathbf{Z}/d\mathbf{Z} \to H_0(\Gamma(n,d), IA_n^{\mathrm{ab}})$$

which satisfy $\mu \circ \tilde{\tau}_n(1) = id$ and $\tilde{\tau}_n(1) \circ \mu = id$. Therefore we see that $\tilde{\tau}_n(1)$ is an isomorphism. From now on, we identify $H_0(\Gamma(n,d), IA_n^{\mathrm{ab}})$ with $(H^* \otimes_{\mathbf{Z}} \Lambda^2 H) \otimes_{\mathbf{Z}} \mathbf{Z}/d\mathbf{Z}$ using this isomorphism.

Now, since the natural map $\rho : \mathrm{Aut}\,F_n \to GL(n, \mathbf{Z})$ is surjective, the restriction map $IA_{n,d} \to \Gamma(n,d)$ of ρ is surjective. Hence we have an exact sequence

$$1 \to IA_n \to IA_{n,d} \to \Gamma(n,d) \to 1.$$

Considering the homological five-term exact sequence of this exact sequence, we have

$$H_2(IA_n, \mathbf{Z}) \to H_2(IA_{n,d}, \mathbf{Z}) \to H_0(\Gamma(n,d), IA_n^{\mathrm{ab}}) \xrightarrow{\iota} IA_{n,d}^{\mathrm{ab}} \to \Gamma(n,d)^{\mathrm{ab}} \to 0.$$

Kawazumi [6] showed that the first Johnson homomorphism $\tau_n(1)$ extends a homomorphis $\tau_{n,d} : IA_{n,d}^{\mathrm{ab}} \to (H^* \otimes_{\mathbf{Z}} \Lambda^2 H) \otimes_{\mathbf{Z}} \mathbf{Z}/d\mathbf{Z}$ such that $\tau_{n,d}(1) \circ \iota = id$. Hence we conclude that the map ι is injective and a short exact sequence

$$0 \to (H^* \otimes_{\mathbf{Z}} \Lambda^2 H) \otimes_{\mathbf{Z}} \mathbf{Z}/d\mathbf{Z} \xrightarrow{\iota} IA_n^{\mathrm{ab}} \to \Gamma(n,d)^{\mathrm{ab}} \to 0$$

splits. This completes the proof of Theorem 1.1. $\square$

At the last of this section, we note the structure of $IA_{n,p}$ for an odd prime integer p. For $n \geq 3$, Lee and Szczarba [7] showed that the abelianization $\Gamma(n,p)^{\mathrm{ab}}$ of the congruence subgroup $\Gamma(n,p)$ of leve p is a $\mathbf{Z}/p\mathbf{Z}$-vector space of dimension $n^2 - 1$. Hence we see that $IA_{n,p}^{\mathrm{ab}}$ is a $\mathbf{Z}/p\mathbf{Z}$-vector space of dimension $\frac{1}{2}(n-1)(n^2+2n+2)$. On the other hand, Frasch [3] showed that the congruence subgroup $\Gamma(2,p)$ is a free group of rank $\alpha(p) = 1 + \frac{(p-1)p(p+1)}{12}$. Furthermore Nielsen [9] showed that $IA_2 = \mathrm{Inn}\,F_2$, where $\mathrm{Inn}\,F_n$ denotes the group of inner automorphisms of F_n. Namely, IA_2 is a free group of rank 2. Hence we see that $IA_{2,p}^{\mathrm{ab}} = \mathbf{Z}^{\oplus\,\alpha(p)} \oplus (\mathbf{Z}/p\mathbf{Z})^{\oplus\,2}$.

3. Some remarks on the (co)homology groups of $IA_{n,p}$

In this section we give some remarks on the (co)homology groups of $IA_{n,p}$ for an odd prime integer p.

Frist, we note that the Lyndon-Hochscild-Serre spectral sequence of an exact sequence

$$1 \to IA_n \to IA_{n,p} \to \Gamma(n,p) \to 1 \tag{1}$$

induces the cohomological five-term exact sequence

$$\begin{aligned} 0 \to H^1(\Gamma(n,p), \mathbf{Z}) \to H^1(IA_{n,p}, \mathbf{Z}) \\ \to H^1(IA_n, \mathbf{Z})^{\Gamma(n,p)} \xrightarrow{\text{tr}} H^2(\Gamma(n,p), \mathbf{Z}) \to H^2(IA_{n,p}, \mathbf{Z}). \end{aligned}$$

Then we have

Proposition 3.1. *For $n \geq 2$, the inflation map $H^2(\Gamma(n,p), \mathbf{Z}) \to H^2(IA_{n,p}, \mathbf{Z})$ is injective.*

Proof. From Lemma 2.1, we see that $H^0(\Gamma(n,p), H^1(IA_n, \mathbf{Z})) = 0$ and hence the transgression tr is a 0-map. Therefore Proposition 3.1 follows. □

Now, Arlettaz [1] showed that $H_2(\Gamma(n,p), \mathbf{Q}) \simeq H_2(SL(n, \mathbf{Z}), \mathbf{Q}) = 0$ for $n \geq 9$. Namely, any element of $H_2(\Gamma(n,p), \mathbf{Z})$ is a torsion element. Using the universal coefficients theorem, we obtain that $H^2(\Gamma(n,p), \mathbf{Z}) \simeq H_1(\Gamma(n,p), \mathbf{Z})$. Hence, from Proposition 3.1, we see that $H_1(\Gamma(n,p), \mathbf{Z}) \subset H^2(IA_{n,p}, \mathbf{Z})$. This shows that $H^2(IA_{n,p}, \mathbf{Z})$ has non-trivial p-torsion elements for $n \geq 9$.

Next we consider the case where $n = 2$. We completely calculate the homology groups of $IA_{2,p}$ with trivial coefficients. First, since the groups IA_2 and $\Gamma(2,p)$ are free groups, considering the Lyndon-Hochscild-Serre spectral sequence of an exact sequence

$$1 \to IA_2 \to IA_{2,p} \to \Gamma(2,p) \to 1,$$

we see that the homological dimension of $IA_{2,p}$ is 2. On the other hand, since the first homology group $H_1(IA_{2,p}, \mathbf{Z})$ is obtained in the previous section, it suffices to calculate the second homology group $H_2(IA_{2,p}, \mathbf{Z})$. Our result is

Proposition 3.2. *For any prime integer p, we have*

$$H_2(IA_{2,p}, \mathbf{Z}) = \mathbf{Z}^{\oplus\,(2\alpha(p)-2)}$$

where $\alpha(p) = 1 + \frac{(p-1)p(p+1)}{12}$.

To prove this Theorem, first, we directly calculate the second cohomology groups of $IA_{2,p}$. Then, using the universal coefficients theorem, we obtain the second homology group of $IA_{2,p}$.

Proposition 3.3. *For any odd prime integer p, we have*

$$H^2(IA_{2,p}, \mathbf{Z}) = \mathbf{Z}^{\oplus\,(2\alpha(p)-2)} \oplus (\mathbf{Z}/p\mathbf{Z})^{\oplus\,2}.$$

Proof. Considering the spectral sequence of the exact sequence (1), we have

$$H^2(IA_{2,p}, \mathbf{Z}) = H^1(\Gamma(2,p), H^1(\operatorname{Inn} F_2, \mathbf{Z})).$$

Let H be the abelianization of F_2 and H^* the dual group of H. We write any element $x \in H$ as a column vector

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in H.$$

Then the congruence subgroup $\Gamma(2,p)$ naturally acts on H and H^* on the left. By an easy argument, we see $H^1(\mathrm{Inn}\, F_2, \mathbf{Z}) \simeq H^*$ as a $\Gamma(2,p)$-module. Since H is $SL(2,\mathbf{Z})$-equivariant isomorphic to H^*, we obtain

$$\begin{aligned} H^1(\Gamma(2,p), H^1(\mathrm{Inn}\, F_2, \mathbf{Z})) &\simeq H^1(\Gamma(2,p), H^*) \\ &\simeq H^1(\Gamma(2,p), H). \end{aligned}$$

Hence it suffices to calculate $H^1(\Gamma(2,p), H)$.

Now we can choose a free basis $\{\gamma_1, \gamma_2, \ldots, \gamma_{\alpha(p)}\}$ of $\Gamma(2,p)$ such that

$$\gamma_1 = \begin{pmatrix} 1 & p \\ 0 & 1 \end{pmatrix}.$$

(See [3].) For any 1-cocycle $f \in Z^1(\Gamma(2,p), H)$, if we put

$$f(\gamma_i) = \begin{pmatrix} x_{i1} \\ x_{i2} \end{pmatrix} \in H$$

for $1 \leq i \leq \alpha(p)$, then we have an isomorphism

$$Z^1(\Gamma(2,p), H) \to \mathbf{Z}^{\oplus\, 2\alpha(p)},$$

$$f \mapsto (x_{i1}, x_{i2}, \ldots, x_{\alpha(p)1}, x_{\alpha(p)2}).$$

Put

$$\gamma_i = \begin{pmatrix} a_i & b_i \\ c_i & d_i \end{pmatrix}$$

for $1 \leq i \leq \alpha(p)$. By the isomorphism above, any 1-coboundary $g \in B^1(\Gamma(2,p), H)$ is mapped to

$$\big((a_i - 1)y_1 + b_i y_2, c_i y_1 + (d_i - 1)y_2\big)_{1 \leq i \leq \alpha(p)}$$

for some

$$y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}.$$

Hence, to calculate $H^1(\Gamma(2,p), H)$, it suffices to find the elementary divisor of a $2 \times 2\alpha(p)$ matrix

$$(2) \qquad \begin{array}{c} \\ \underline{y_1} \\ \underline{y_2} \end{array} \begin{array}{c} \begin{array}{cccccccc} \underline{x_{11}} & \underline{x_{12}} & \cdots & \underline{x_{i1}} & \underline{x_{i2}} & \cdots & \underline{x_{\alpha(p)1}} & \underline{x_{\alpha(p)2}} \end{array} \\ \begin{pmatrix} 0 & 0 & \cdots & a_i - 1 & c_i & \cdots & a_{\alpha(p)} - 1 & c_{\alpha(p)} \\ p & 0 & \cdots & b_i & d_i - 1 & \cdots & b_{\alpha(p)} & d_{\alpha(p)} - 1 \end{pmatrix}. \end{array}$$

Lemma 3.1. *The greatest common divisor of all entries of the first row of the matrix* (2) *is* p.

Proof of Lemma 3.1. Let

$$t = \gcd\big\{a_i - 1, c_i \,\big|\, 1 \leq i \leq \alpha(p)\big\}.$$

We may assume $t > 0$. Since t divides c_i for any i, and $\{\gamma_1, \ldots, \gamma_{\alpha(p)}\}$ is a generator of $\Gamma(2,p)$, for any element

$$\sigma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma(2,p),$$

t divides c. On the other hand, since there exists

$$\begin{pmatrix} 1 & 0 \\ p & 1 \end{pmatrix} \in \Gamma(p),$$

it shows that t divides p. Since $p|t$, we have $t = p$. This completes the proof of the lemma.

By this lemma, we can transform the matrix (2) into

$$\begin{array}{c} \\ \underline{y_1} \\ \underline{y_2} \end{array} \begin{array}{c} \begin{array}{ccccccc} \underline{x_{11}} & \underline{x_{12}} & \underline{x_{21}} & \cdots & \cdots & \underline{x_{\alpha(p)1}} & \underline{x_{\alpha(p)2}} \end{array} \\ \begin{pmatrix} p & 0 & 0 & \cdots & \cdots & 0 & 0 \\ 0 & p & 0 & \cdots & \cdots & 0 & 0 \end{pmatrix} \end{array}$$

using elementary transformations. This shows $H^2(IA_{2,p}, \mathbf{Z}) = \mathbf{Z}^{\oplus\,(2\alpha(p)-2)} \oplus (\mathbf{Z}/p\mathbf{Z})^{\oplus\,2}$.
□

Similarly, we obtain the following results. The proof is left to the reader.

Proposition 3.4. *For any odd prime integer p and an integer $q \geq 2$, we have*

$$H^2(IA_{2,p}, \mathbf{Z}/q\mathbf{Z}) \simeq \begin{cases} (\mathbf{Z}/q\mathbf{Z})^{\oplus(2\alpha(p)-2)} & \text{if } (q,p) = 1, \\ (\mathbf{Z}/q\mathbf{Z})^{\oplus(2\alpha(p)-2)} \oplus (\mathbf{Z}/p\mathbf{Z})^{\oplus 2} & \text{if } q = p^e. \end{cases}$$

Using Propositions 3.3 and 3.4, we obtain the second homology group $H_2(IA_{2,p}, \mathbf{Z})$ by the universal coefficients theorem. This completes the proof of Theorem 3.2.

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