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Abstract

The author gives a simple proof of the representation theorem for law invariant convex risk measures which was obtained by Kusuoka [6], Frittelli-Gianin [3] and Jouini- Schachermayer-Touzi [5].

1 Introduction

The idea of coherent risk measures has been introduced by Artzner, Delbaen, Eber and Heath [1]. Then Föllmer and Scheid [4] extended this notion to convex risk measures. Let me introduce the definition of convex risk measures first.

Let $(\Omega, \mathcal{F}, P)$ be a probability space. We denote $L^\infty(\Omega, \mathcal{F}, P)$ by L^∞.

Definition 1 *We say that a map $\rho : L^\infty \to \mathbf{R}$ is a convex risk measure if the following are satisfied.*
(1) $\rho(0) = 0$.
(2) *For any $c \in \mathbf{R}$ and $X \in L^\infty$, we have*

$$\rho(X + c) = \rho(X) - c.$$

(3) *If $X \geqq Y$, $X, Y \in L^\infty$, then $\rho(X) \leqq \rho(Y)$.*
(4) *For any $\lambda \in [0, 1]$, and $X, Y \in L^\infty$,*

$$\rho(\lambda X + (1 - \lambda)Y) \leqq \lambda\rho(X) + (1 - \lambda)\rho(Y).$$

Also, we introduce the following notion.

Definition 2 *We say that a convex risk measure $\rho : L^\infty \to \mathbf{R}$ is law invariant, if $\rho(X) = \rho(Y)$ for any $X, Y \in L^\infty$ with the same probability laws.*

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Let $\mathcal{D}$ be the set of probability distribution functions of bounded random variables, i.e., $\mathcal{D}$ is the set of non-decreasing right-continuous functions F on $\mathbf{R}$ such that there are $z_0, z_1 \in \mathbf{R}$ for which $F(z) = 0$, $z < z_0$ and $F(z) = 1$, $z \geqq z_1$. Let us define $Z : [0,1) \times \mathcal{D} \to \mathbf{R}$ by

$$Z(x, F) = \inf\{z; F(z) > x\}, \qquad x \in [0,1), \ F \in \mathcal{D}.$$

$Z(x, F)$ is a version of $F^{-1}(x)$. $Z(\cdot, F) : [0,1) \to \mathbf{R}$ is a non-decreasing and right continuous function, and the probability distribution function of $Z(x, F)$ under the Lebesgue measure dx on $[0,1)$ is F. We denote by F_X the probability distribution function of a random variable X.

For each $\alpha \in (0,1]$, let $\rho_\alpha : L^\infty \to \mathbf{R}$ be given by

$$\rho_\alpha(X) = -\alpha^{-1} \int_0^\alpha Z(x, F_X) dx, \qquad X \in L^\infty.$$

Also, we define $\rho_0 : L^\infty \to \mathbf{R}$ by

$$\rho_0(X) = -Z(0, F_X) = -ess.inf\ X \qquad X \in L^\infty.$$

Then it is easy to see that $\rho_\cdot(X) : [0,1] \to \mathbf{R}$ is a non-increasing continuous function for any $X \in L^\infty$.

Let $\mathcal{M}_{[0,1]}$ be the set of probability measures on $[0,1]$.

Then combining the results by [6], Frittelli-Gianin [3] and Jouini- Schachermayer-Touzi [5], we have the following.

Theorem 3 *Assume that $(\Omega, \mathcal{F}, P)$ is a standard atomless probability space. Let $\rho : L^\infty \to \mathbf{R}$. Then the following conditions are equivalent.*
(1) *There is a subset $\mathcal{A}$ of the set $\mathcal{M}_{[0,1]} \times \mathbf{R}$ such that*

$$\sup\{b;\ (m, b) \in \mathcal{A}\} = 0$$

and

$$\rho(X) = \sup\{\int_{[0,1]} \rho_\alpha(X) m(d\alpha) + b;\ (m, b) \in \mathcal{A}\}, \qquad X \in L^\infty.$$

(2) *ρ is a law invariant convex risk measure.*

Our purpsoe of the present paper is to give a simple and direct proof for this Theorem.

Remark 4 *One can easily prove that*

$$\rho_\alpha(X) = -\inf\{E[gX];\ g \in L^\infty,\ 0 \leqq g \leqq \frac{1}{\alpha},\ E[g] = 1\}, \quad X \in L^\infty$$

for any $\alpha \in (0,1]$. Here we do not have to assume that $(\Omega, \mathcal{F}, P)$ is atomless. So we can easily check that ρ_α, $\alpha \in [0,1]$, are law invariant convex risk measures. Therefore it is easy to prove that the condition (1) implies the condition (2) in Theorem 3.

2 Preparations

Let $N \geqq 2$. In this section, we consider a probability space $(\Omega_N, \mathcal{G}_N, P_N)$ such that $\Omega_N = \{1, \ldots, N\}$, $\mathcal{G}_N$ be the set of subsets of Ω_N, and $P_N(\{\omega\}) = \frac{1}{N}$, $\omega \in \Omega_N$.

Our aim in this section is to prove the following.

Theorem 5 *Let $\rho : L^\infty \to \mathbf{R}$. Then the following conditions are equivalent.*
(1) *There is a subset $\mathcal{A}_0$ of the set $\mathcal{M}_{[0,1]} \times \mathbf{R}$ such that*

$$\sup\{b;\ (m, b) \in \mathcal{A}_0\} = 0$$

and

$$\rho(X) = \sup\{\int_{[0,1]} \rho_\alpha(X) m(d\alpha) + b;\ (m, b) \in \mathcal{A}_0\}, \qquad X \in L^\infty.$$

(2) *ρ is a law invariant convex risk measure.*

By Remark 4, it is sufficient to prove that the condition (2) implies the condition (1). So we prove the converse. Let ρ is a law invariant convex risk measure and let $\mathcal{C}$ be a subset of $L^\infty \times \mathbf{R}$ given by

$$\mathcal{C} = \{(a, b) \in L^\infty \times \mathbf{R};\ \rho(X) \geqq -\sum_{i=1}^N a(i)X(i) + b \text{ for all } X \in L^\infty\}.$$

Since ρ is a concave function defined in L^∞ and L^∞ is finite dimensional, we see that

$$\rho(X) = \sup\{-\sum_{i=1}^N a(i)X(i) + b;\ (a, b) \in \mathcal{C}\}, \qquad X \in L^\infty. \tag{1}$$

Moreover, we have the following.

Proposition 6 *For any $(a, b) \in \mathcal{C}$, we have the following.*
(1) $a(i) \geqq 0, \qquad i = 1, \ldots, N.$
(2) $\sum_{i=1}^N a(i) = 1.$

Proof. Let $e_i \in L^\infty$, $i = 1, \ldots, N$, such that $e_i(i) = 1$, and $e_i(j) = 0$, $j \neq i$. Then we have for any $c > 0$

$$0 \leqq -c^{-1}\rho(ce_i) \leqq a(i) - c^{-1}b$$

Lettig $c \to \infty$, we have the assertion (1).

Note that for any $c \in \mathbf{R}$, we have

$$0 = -\rho(c) - c \leqq c(\sum_{i=1}^N a(i) - 1) - b$$

So we have for any $c > 0$

$$(\sum_{i=1}^N a(i) - 1) - c^{-1}b \leqq 0 \text{ and } (\sum_{i=1}^N a(i) - 1) + c^{-1}b \geqq 0.$$

Lettig $c \to \infty$, we have the assertion (2). ∎

Let $\mathcal{S}_N$ be the set of permutations on Ω_N. Then for any $a \in L^\infty$, there is a $\sigma_a \in \mathcal{S}_N$ such that

$$a(\sigma_a(N)) \leqq a(\sigma(N-1)) \leqq \cdots \leqq a(\sigma_a(1))$$

Then we have the following.

Proposition 7 (1) *For any $(a,b) \in \mathcal{C}$, and $\sigma \in \mathcal{S}_N$, $(a \circ \sigma, b) \in \mathcal{C}$.*
(2) *Let $(a,b) \in \mathcal{C}$ and let m_a be a measure on $[0,1]$ be given by*

$$m_a(\{\frac{j}{N}\}) = (a(\sigma_a(j)) - a(\sigma_a(j+1)))j, \qquad j = 1,\dots,N-1,$$

$$m_a(\{1\}) = a(\sigma_a(N))N \quad \textit{and} \quad m_a([0,1] \setminus \{\frac{1}{N}, \frac{2}{N}, \dots, 1\}) = 0.$$

Then $m_a \in \mathcal{M}_{[0,1]}$ and

$$\max\{-\sum_{i=1}^N (a \circ \sigma)(i) X(i);\ \sigma \in \mathcal{S}_N\} = \int_{[0,1]} \rho_\alpha(X) m_a(dx), \qquad X \in L^\infty.$$

Proof. Let $X \in L^\infty$. Then it is obvious that random variables X and $X \circ \sigma^{-1}$ has the same probability law. Therefore we have

$$\rho(X) = \rho(X \circ \sigma^{-1}) \geqq -\sum_{i=1}^N a(i) X(\sigma^{-1}(i)) + b = -\sum_{i=1}^N a(\sigma(i)) X(i) + b.$$

This implies the assertion (1).

Now we will prove the assertion (2). Let $X \in L^\infty$. Then there is an $\tau_X \in \mathcal{S}_N$ such that

$$X(\tau_X(1)) \leqq X(\tau_X(2)) \leqq \cdots \leqq X(\tau_X(N)).$$

It is easy to see that

$$X(\tau_X(k)) = N \int_{(k-1)/N}^{k/N} Z(x; F_X) dx, \qquad k = 1,\dots,N,$$

an so

$$\sum_{j=1}^k X(\tau_X(j)) = -k\rho_{k/N}(X), \qquad k = 1,\dots,N.$$

Then we have

$$\sum_{i=1}^N a(i) X(i) = \sum_{i=1}^N a(\sigma_a(i)) X(\sigma_a(i))$$

$$= \sum_{i=1}^N (a(\sigma_a(N)) + a(\sigma_a(i) - a(\sigma_a(N))) X(\sigma_a(i))$$

$$= a(\sigma_a(N))(\sum_{i=1}^N X(\sigma_a(i))) + \sum_{i=1}^{N-1} (\sum_{j=i+1}^N (a(\sigma_a(j-1) - a(\sigma_a(j))) X(\sigma_a(i))$$

$$= a(\sigma_a(N))(\sum_{i=1}^{N} X(\sigma_a(i))) + \sum_{j=2}^{N}(\sum_{i=1}^{j-1} X(\sigma_a(i)))(a(\sigma_a(j-1) - a(\sigma_a(j)))$$

$$\geqq a(\sigma_a(N))(\sum_{i=1}^{N} X(\tau_X(i))) + \sum_{j=2}^{N}(\sum_{i=1}^{j-1} X(\tau_X(i)))(a(\sigma_a(j-1) - a(\sigma_a(j)))$$

$$= -\int_{[0,1]} \rho_\alpha(X) m_a(d\alpha).$$

Note that

$$\sum_{i=1}^{N} a(\sigma_a \circ \tau_X^{-1}(i))X(i) = \sum_{i=1}^{N} a(i)X(\tau_X \circ \sigma_a^{-1}(i)) = -\int_{[0,1]} \rho_\alpha(X) m_a(d\alpha).$$

So letting $X = 1$, we see that $m_a([0,1]) = 1$. These also imply the assertion (2). ∎

Now let

$$\mathcal{A}_0 = \{(m_a, b) \in \mathcal{M}_{[0,1]} \times \mathbf{R};\ (a,b) \in \mathcal{C}\}$$

Then we see from Equation (1) and Proposition 7, that the condition (1) is satisfied for this $\mathcal{A}_0$. This completes the proof of Theorem 5.

3 Proof of Theorem 3

By Remark 4, it is sufficient to prove that the condition (2) implies the condition (1).

Let ρ is a law invariant convex risk measure, and let

$$\mathcal{A} = \Big\{(m,b) \in \mathcal{M}_{[0,1]} \times \mathbf{R}\ ;\ \rho(X) \geqq \int_{[0,1]} \rho_\alpha(X) m(d\alpha) + b, \quad \text{for all } X \in L^\infty(\Omega)\Big\}.$$

Then it is sufficient to prove the following.

$$\rho(X) \leqq \sup\Big\{\int_{[0,1]} \rho_\alpha(X) m(d\alpha) + b\ ;\ (m,b) \in \mathcal{A}\Big\}. \tag{2}$$

Since $(\Omega, \mathcal{F}, P)$ is atomless standard probability space, we may think that $\Omega = [0,1)$, $\mathcal{F} = \mathcal{B}\big([0,1)\big)$, and P is a Lebesgue measure on $[0,1)$. For any $n \geqq 1$, let

$$\mathcal{F}_n = \sigma\big\{1_{[(k-1)2^{-n}, k2^{-n})}\ ;\ k = 1, 2, \ldots, 2^n\big\}.$$

Then we see that

$$\mathcal{F}_1 \subset \mathcal{F}_2 \subset \mathcal{F}_3 \subset \cdots \quad \text{and} \quad \sigma\Big(\bigcup_{n=1}^{\infty} \mathcal{F}_n\Big) = \mathcal{F}.$$

Let

$$\mathcal{A}_n = \Big\{(m,b) \in \mathcal{M}_{[0,1]} \times \mathbf{R}\ ;\ \rho(X) \geqq \int_{[0,1]} \rho_\alpha(X) m(d\alpha) + b \ \text{ for all } X \in L^\infty(\Omega, \mathcal{F}_n, P)\Big\}.$$

Then we have

$$\mathcal{A}_1 \supset \mathcal{A}_2 \supset \mathcal{A}_3 \supset \cdots \supset \mathcal{A}.$$

Note that $\mathcal{M}_{[0,1]}$ is a compact subset of the dual space of $C([0,1];\mathbf{R})$ with weak * topology. Since $\rho_{\cdot}(X) : [0,1] \longrightarrow \mathbf{R}$ is continuous for all $X \in L^\infty$, $\mathcal{A}$, $\mathcal{A}_n$, $n = 1, 2, \ldots$, are closed in $\mathcal{M}_{[0,1]} \times \mathbf{R}$.

Proposition 8 *Let* $\mathcal{A}_\infty = \bigcap_{n=1}^\infty \mathcal{A}_n$. *Then* $\mathcal{A}_\infty = \mathcal{A}$.

Proof. Let $(m,b) \in \mathcal{A}_\infty$. Let $X \in L^\infty(\Omega, \mathcal{F}, P)$, and fix it. Let $Y \in L^\infty$ be given by $Y(\omega) = Z(\omega; F_X)$, $\omega \in \Omega = [0,1)$. Since random variables X and Y have the same probability law, we see that $\rho(X) = \rho(Y)$. Let Y_n, $n = 1, 2, \ldots$, be random variables given by

$$Y_n(\omega) = Z(\frac{k}{2^n}-; F_X), \qquad \frac{k-1}{2^n} \leqq \omega < \frac{k}{2^n},\ k = 1, 2, \ldots, 2^n.$$

Then we see that $Y_n(\omega) \downarrow Y(\omega)$, for any $\omega \in \Omega$. Since $(m,b) \in \mathcal{A}_n$, $n \geq 1$, we have

$$\rho(Y) \geqq \rho(Y_n) \geqq \int_{[0,1]} \rho_\alpha(Y_n) m(d\alpha) + b, \qquad \alpha \in [0,1].$$

It is easy to see that $\rho_\alpha(\mu_{Y_n}) \uparrow \rho_\alpha(Y)$, and so we have

$$\rho(X) = \rho(Y) \geqq \int_{[0,1]} \rho_\alpha(Y) m(d\alpha) + b = \int_{[0,1]} \rho_\alpha(X) m(d\alpha) + b.$$

This implies that $\mathcal{A}_\infty \subset \mathcal{A}$. It is obvious that $\mathcal{A}_\infty \supset \mathcal{A}$, and so we have the assertion. ∎

Now let us prove Theorem 3. For each $W \in L^\infty(\Omega_{2^n}, \mathcal{G}_{2^n}, P_{2^n})$, let $U_n(W) : \Omega \to \mathbf{R}$ be given by

$$U_n(W)(\omega) = \sum_{k=1}^{2^n} W(k) 1_{[(k-1)2^{-n}, k2^{-n})}(\omega).$$

Then $U_n : L^\infty(\Omega_{2^n}, \mathcal{G}_{2^n}, P_{2^n}) \to L^\infty(\Omega, \mathcal{F}_n, P)$ is bijective. Let $\rho_n : L^\infty(\Omega_{2^n}, \mathcal{G}_{2^n}, P_{2^n}) \to \mathbf{R}$ be defined by $\rho_n(W) = \rho(U_n(W))$. Then it is easy to see that ρ_n is law invariant, convex risk measure and that

$$\rho_n(W) \geqq \int_{[0,1]} \rho_\alpha(W) m(d\alpha) + b, \qquad W \in L^\infty(\Omega_{2^n}, \mathcal{G}_{2^n}, P_{2^n})$$

if and only if

$$\rho(X) \geqq \int_{[0,1]} \rho_\alpha(X) m(d\alpha) + b, \qquad X \in L^\infty(\Omega, \mathcal{F}_n, P)$$

for any $(m,b) \in \mathcal{M}_{[0,1]} \times \mathbf{R}$. This observation and Theorem 5 show that

$$\rho(X) = \inf\left\{ \int_{[0,1]} \rho_\alpha(X) m(d\alpha) + b\ ;\ (m,b) \in \mathcal{A}_n \right\}, \quad X \in L^\infty(\Omega, \mathcal{F}_n, P). \tag{3}$$

Let us take an arbitrary $X \in L^\infty(\Omega, \mathcal{F}, P)$ and fix it. Let Y and $\tilde{Y}_n$, $n = 1, 2, \ldots$, be random variables given by $Y(\omega) = Z(\omega, F_X)$, $\omega \in [0,1)$, and

$$\tilde{Y}_n(\omega) = Z(\frac{k-1}{2^n} \vee 0; F_X), \quad \frac{k-1}{2^n} \leqq \omega < \frac{k}{2^n}, k = 1, 2, \ldots, 2^n.$$

Then we see that

$$Z(x; F_{\tilde{Y}_n}) = \tilde{Y}_n(x) \uparrow Y(x-), \qquad x \in (0,1),$$

and so we see that

$$\rho_\alpha(\tilde{Y}_n) \downarrow \rho_\alpha(Y) = \rho_\alpha(X), \quad n \to \infty, \qquad \alpha \in (0,1].$$

Also, we see that

$$\rho_0(\tilde{Y}_n) = -\tilde{Y}_n(0) = -Y(0) = \rho_0(X)$$

So we see that $\rho_\alpha(\tilde{Y}_n)$ converges to $\rho_\alpha(X)$ uniformly in $\alpha \in [0,1]$.

Since $\tilde{Y}_n \in L^\infty(\Omega, \mathcal{F}_n, P)$, we see from Equation (2) that there exists $(m_n, b_n) \in \mathcal{A}_n$, for each $n \geq 1$, such that

$$\rho(\tilde{Y}_n) \leqq \int_{[0,1]} \rho_\alpha(\tilde{Y}_n) m_n(d\alpha) + b_n + \frac{1}{n}.$$

Note that

$$0 = \rho(0) \geqq \int_{[0,1]} \rho_\alpha(0) m_n(d\alpha) + b_n = b_n$$

and that

$$-||\tilde{Y}_n||_\infty = \rho(||\tilde{Y}_n||_\infty) \leqq \rho(\tilde{Y}_n) \leqq \int_{[0,1]} \rho_\alpha(\tilde{Y}_n) m_n(d\alpha) + b_n + \frac{1}{n} \leqq ||\tilde{Y}_n||_\infty + b_n + 1.$$

So we have

$$0 \geqq b_n \geqq -2||\tilde{Y}_n||_\infty - 1 \geqq -2||X||_\infty + 1.$$

Since $\mathcal{M}_{[0,1]}$ is compact, there are a subsequence $\{n_k; k = 1, 2, \ldots\}$ and $(m, b) \in \mathcal{M}_{[0,1]} \times \mathbf{R}$ such that

$$(m_{n_k}, b_{n_k}) \to (m, b), \quad n \to \infty, \qquad \text{in } \mathcal{M}_{[0,1]} \times \mathbf{R}.$$

It is obvious that $(m, b) \in \mathcal{A}_{n_k}$, $k = 1, 2, \ldots$, and so we see that $(m, b) \in \mathcal{A}_\infty$. Also we have

$$\int_{[0,1]} \rho_\alpha(\tilde{Y}_{n_k}) m_{n_k}(d\alpha) \to \int_{[0,1]} \rho_\alpha(Y) m(d\alpha).$$

On the other hand, we see that

$$\rho(\tilde{Y}_n) \geqq \rho(Y) = \rho(X)$$

So we see that

$$\rho(X) \leqq \int_{[0,1]} \rho_\alpha(X) m(d\alpha) + b.$$

This proves that

$$\rho(X) \leqq \sup\{\int_{[0,1]} \rho_\alpha(X) m(d\alpha) + b;\ (m, b) \in \mathcal{A}\}.$$

This completes the proof of Theorem 3.

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