

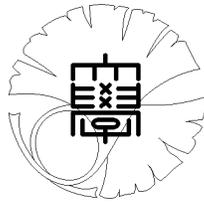
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**Acyclic polarizations and localization of
Riemann-Roch numbers I**

by

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Acyclic polarizations and localization of Riemann-Roch numbers I

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Abstract

We define a local Riemann-Roch number for an open symplectic manifold when a complete integrable system without Bohr-Sommerfeld orbit is provided on its end.

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1 Introduction

The purpose of this paper is to give a localization technique for the index of $Spin^c$ Dirac operator. Our localization makes use of no group action, but a family of acyclic flat connections on tori. A typical example is given by a symplectic manifold with an integrable system.

For a complete integrable system on a closed symplectic manifold, the Riemann-Roch number is sometimes equal to the number of Bohr-Sommerfeld orbits. D. Borthwick, T. Paul and A. Uribe gave a precise relation between Bohr-Sommerfeld orbits and the kernels of twisted $Spin^c$ Dirac operators using Fourier integral operators [1]. A problem here would be how to count singular Bohr-Sommerfeld orbits, and how to count the contribution from other singular orbits if there are. M. D. Hamilton gave an approach to deal with the singular Bohr-Sommerfeld orbits using J. Śniatycki's framework [8].

When an integrable system is associated to a Hamiltonian group action, the geometric quantization conjecture of Guillemin and Sternberg is a localization property of the Riemann-Roch character. H. Duistermaat, V.

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Guillemin, E. Meinrenken and S. Wu gave a proof of the conjecture for torus actions using the geometric localization provided by E. Lerman's symplectic cuts [2] [9]. Y. Tian and W. Zhang gave a proof using a direct analytic localization with perturbation of the Dirac operator [10].

In this paper we give a localization property of the Riemann-Roch numbers for complete integrable system possibly with singular orbits. We define multiplicities for Bohr-Sommerfeld orbits and singular orbits so that the total sum of the multiplicities for all the fibers is exactly equal to the Riemann-Roch number. Our method is flexible and allows various generalizations. In this paper, however, we explain only the simplest case. In our subsequent paper we will explain some of the generalizations and, as an application, an approach to the Guillemin-Sternberg conjecture for Hamiltonian torus actions [3].

Our idea is simple. Let M be a $2n$ -dimensional closed symplectic manifold with a prequantum line bundle L . Suppose X is an n -dimensional affine space, and $\pi : M \rightarrow X$ a completely integrable system. The symplectic structure ω gives rise to an element $[D \otimes L]$ of the K-homology group $K_0(M)$, where D is the Dolbeault operator for an almost complex structure compatible with ω . The projection π gives an element of $K_0(X)$ defined to be the pushforward $\pi_*[D \otimes L]$. The Riemann-Roch number is calculated as the further pushforward of $\pi_*[D \otimes L]$ with respect to the map from X to a point.

If we have a formulation of K-homology group in terms of some geometric data, and if a geometric representative of $\pi_*[D \otimes L]$ has a *localized support*, then the Riemann-Roch number can be calculated by the data on the support.

In fact it is possible to realize this idea rigorously if we use a formulation of K-homology in terms of some notion of generalized vector bundle with Clifford module bundle action, which is developed in [5], [7] ¹. In this short paper, however, we explain only the localization property without appealing to the framework of [5] at the expense of giving up identifying $\pi_*[D \otimes L]$ in terms of geometric data.

Some key points of our arguments are:

1. The restriction of the Dolbeault operator, or the spin^c Dirac operator, on a Lagrangian submanifold is equal to its de Rham operator at least on the level of a principal symbol.

¹Strictly speaking what we actually need here is the notion of a *K-cohomology cocycle with local coefficient* which is defined using some action of a Clifford algebra bundle. For a manifold if the Clifford algebra module is the one generated by its tangent space, the twisted K-cohomology group is identified with its K-homology through a duality.

2. A flat line bundle over a torus is trivial if and only if the corresponding twisted de Rham complex is acyclic.
3. The localization we use is given by an adiabatic limit.
4. The Laplacian corresponding to the de Rham operator along Lagrangian fibers plays the role of the potential term of the spin^c Dirac operator on the base manifold.
5. Since the Laplacian is second-order elliptic differential operator, when it is strictly positive, it can absorb the effect of first-order term.

The last property makes our construction rather flexible.

The organization of this paper is as follows. In Sections 2, 3 and 4, we give localization properties for symplectic setting, topological setting, and analytical setting respectively. The latter is a generalization of the former for each stage. In Section 5 we prove the localization. In Section 6 we give an example for 2-dimensional topological case. Finally in Section 7 we give comments for possible generalizations.

2 Symplectic formulation

Let M be a $2n$ -dimensional closed symplectic manifold with a symplectic form ω . Suppose L is a complex Hermitian line bundle over M with Hermitian connection ∇ satisfying that the curvature of ∇ is equal to ω . M has an almost complex structure compatible with ω , and we can define the Riemann-Roch number $RR(M, L)$ as the index of the Dolbeault operator with coefficients in L .

Let X be an n -dimensional affine space, and $\pi : M \rightarrow X$ a complete integrable system. Then generic fibers of π are disjoint unions of finitely many n -dimensional tori with canonical affine structures.

Definition 2.1 $x \in X$ is L -acyclic if x is a regular value of π and L does not have any non-vanishing parallel section over the fiber at x .

The main purpose of this paper is to show that $RR(M, L)$ is localized at non L -acyclic points. More precisely

Theorem 2.2 *Suppose $X = U_\infty \cup (\cup_{i=1}^m U_m)$ is an open covering satisfying the following properties.*

1. $\{U_i\}_{i=1}^m$ are mutually disjoint.

2. U_∞ consists of L -acyclic points.

Let $V_i = \pi^{-1}(U_i)$. Then for each $i = 1, \dots, m$ there exists an integer $RR(V_i, L)$, which depends only on the data restricted on U_i , such that

$$RR(M, L) = \sum_{i=1}^m RR(V_i, L).$$

Here the integer $RR(V_i, L)$ is invariant under continuous deformations of the data.

Remark 2.3 The theorem asserts that the Riemann-Roch number $RR(M, L)$ is localized at non-singular Bohr-Sommerfeld fibers and singular fibers.

Remark 2.4 While the Riemann-Roch number $RR(M, L)$ for the closed symplectic manifold M depends only on the symplectic structure ω , the localized Riemann-Roch number $RR(V_i, L)$ depends on the restrictions of ω, π, ∇ as well, though we omit this dependence in the notation if there is no confusion.

In the next section we reduce the above localization theorem to a slightly more general localization (Theorem 3.3) formulated purely topologically.

3 Topological formulation

In this section let M be a $2n$ -dimensional closed spin^c manifold,² and E a complex Hermitian vector bundle over M . We define the Riemann-Roch number $RR(M, E)$ as the index of the spin^c Dirac operator with coefficients in E .³ Let V be an open subset of M .

Definition 3.1 A real polarization on V is the data (U, π, ϕ, J) satisfying the following properties.

1. U is an n -dimensional smooth manifold.
2. $\pi : V \rightarrow U$ is a fiber bundle whose fibers are disjoint unions of finitely many n -dimensional tori with affine structures.

²In this paper we take a convention of spin^c -structures which do not need any Riemannian metrics. See Appendix A for the convention.

³Precisely, in order to define the spin^c Dirac operator with coefficients in E we need a unitary connection on E . But it is well-known that $RR(M, E)$ itself does not depend on the choice of connections. So we do not mention it here.

3. $\phi : \pi_* T_{fiber} V \rightarrow TU$ is an isomorphism between two real vector bundles, where $\pi_* T_{fiber} V$ is the vector bundle on U consisting of parallel sections of the tangent bundle of the fiber $\pi^{-1}(x)$ for each $x \in U$.
4. J is an almost complex structure on V which is a reduction of the given spin^c -structure.
5. The composition of $J : T_{fiber} V \rightarrow TV$ and $\pi_* : TV \rightarrow TU$ is equal to the map induced from ϕ .

Suppose that $V \subset M$ has a real polarization (U, π, ϕ, J) such that the restriction $E|_V$ has a unitary connection ∇ along fibers for the bundle structure $\pi : V \rightarrow U$.

Definition 3.2 (E, ∇) is *acyclic* if the restriction $(E, \nabla)|_{\pi^{-1}(x)}$ is a flat vector bundle and the twisted de Rham cohomology group $H^*(\pi^{-1}(x), (E, \nabla)|_{\pi^{-1}(x)})$ is zero for every $x \in U$.

Theorem 3.3 *Let M be a closed spin^c manifold and E a complex Hermitian vector bundle over M . Suppose $M = V_\infty \cup (\cup_{i=1}^m V_m)$ is an open covering satisfying the following properties.*

1. $\{V_i\}_{i=1}^m$ are mutually disjoint.
2. V_∞ has a real polarization (U, π, ϕ, J) .
3. $E|_{V_\infty}$ has a unitary connection ∇ along fibers for the bundle structure $\pi : V_\infty \rightarrow U$.
4. $(E|_{V_\infty}, \nabla)$ is *acyclic*.

Then for each $i = 1, \dots, m$ there exists an integer $RR(V_i, E)$, which depends only on the data of the polarization and ∇ restricted on $V_i \cap V_\infty$, such that

$$RR(M, E) = \sum_{i=1}^m RR(V_i, E).$$

Here the integer $RR(V_i, E)$ is invariant under continuous deformations of the data.

Proof of Theorem 2.2 assuming Theorem 3.3. Note that the symplectic structure gives an isomorphism $T^*U_\infty \cong \pi_* T_{fiber} V_\infty$. Fix a Riemannian metric on TU_∞ so that we have an isomorphism $TU_\infty \cong T^*U_\infty$. Define ϕ by using these two. By fixing a splitting $TV_\infty \cong T_{fiber} V_\infty \oplus \pi^* TU_\infty$, ϕ induces an almost complex structure, which determine the spin^c structure. We take E to be L . Then the rest would be obvious. \square

In the next section we further reduce Theorem 3.3 to more general localization for some Dirac type operator (Theorem 4.4).

4 Analytical formulation

In this section let M be a closed Riemannian manifold. We denote by $Cl(TM)$ the Clifford algebra bundle over M generated by TM . Let $W = W^0 \oplus W^1$ be a $\mathbb{Z}/2\mathbb{Z}$ -graded complex Hermitian vector bundle over M with a structure of $Cl(TM)$ -module such that for each vector v in TM , the action of v is skew-Hermitian and of degree-one.

Definition 4.1 Using a Dirac type operator on W whose symbol is given by the Clifford action, the integer $Ind(M, W)$ is defined to be the index of the Dirac type operator.

Let V be an open subset of M .

Definition 4.2 A *generalized real polarization* on V is the data (U, π, D_{fiber}) satisfying the following properties.

1. U is a Riemannian manifold.
2. $\pi : V \rightarrow U$ is a fiber bundle with fiber a closed manifold.
3. Let $TV = T_{fiber}V \oplus T_{fiber}^\perp V$ be the orthogonal decomposition with respect to the Riemannian metric on V . Then the projection gives an isometric isomorphism $T_{fiber}^\perp V \cong \pi^*TU$.
4. $D_{fiber} : \Gamma(W|_V) \rightarrow \Gamma(W|_V)$ is a family of Dirac-type operators along fibers anti-commuting with the Clifford action of TU in the following sense.
 - (a) D_{fiber} is an order-one, formally self-adjoint differential operator of degree-one.
 - (b) D_{fiber} contains only the derivatives along fibers, i.e, D_{fiber} commutes with multiplication of the pullback of smooth functions on U .
 - (c) The principal symbol of D_{fiber} is given by the Clifford action of $T_{fiber}V$.
 - (d) The Clifford action of TU on $W|_V$ anti-commutes with D_{fiber} . Here the Clifford action of TU on $W|_V$ is defined through the horizontal lift $\pi^*TU \rightarrow TV$ with respect to the Riemannian metric on V .

Definition 4.3 A generalized real polarization (U, π, D_{fiber}) on V is *acyclic* if for each $x \in U$, the restriction of D_{fiber} on $\Gamma(W|_{\pi^{-1}(x)})$ has zero kernel.

Theorem 4.4 Let M be a closed Riemannian manifold and $W = W^0 \oplus W^1$ a $Cl(TM)$ -module bundle as above. Suppose $M = V_\infty \cup (\cup_{i=1}^m V_i)$ is an open covering satisfying the following properties.

1. $\{V_i\}_{i=1}^m$ are mutually disjoint.
2. V_∞ has an acyclic generalized real polarization (U, π, D_{fiber}) .

Then for each $i = 1, \dots, m$ there exists an integer $Ind(V_i, W)$, which depends only on the data of the acyclic real polarization restricted on $V_i \cap V_\infty$, such that

$$Ind(M, W) = \sum_{i=1}^m Ind(V_i, W).$$

Here the integer $Ind(V_i, W)$ is invariant under continuous deformations of the data.

The proof of Theorem 3.3 follows from the next obvious lemma.

Lemma 4.5 Let T be an n -dimensional torus with an affine structure. Let \mathfrak{X} be the n -dimensional vector space of parallel vector fields. For any Euclidean metric on \mathfrak{X} , T has an induced flat Riemannian metric. Then each element of the dual space \mathfrak{X}^* gives a harmonic 1-form.

Proof of Theorem 3.3 assuming Theorem 4.4. Fix a Riemannian metric on U . Combining it with the flat metric associated with the affine structures on fibers of V_∞ via the almost complex structure J , we define a Riemannian metric on V_∞ and extend it to M . Take W to be the tensor product of the spinor bundle of the spin^c -manifold M and E . Then we define the Dirac operator acting on $\Gamma(W|_{V_\infty})$ by $D_{fiber} := d_{E, fiber} + d_{E, fiber}^*$, where $d_{E, fiber}$ is the exterior derivative on fibers twisted by the unitary connection ∇ on E and $d_{E, fiber}^*$ is its formal adjoint. The above lemma implies the anti-commutativity between D_{fiber} and the Clifford action of TU . The acyclic condition for (E, ∇) implies the acyclicity for (U, π, D_{fiber}) . \square

5 Local index

Let M be a Riemannian manifold and $W = W^0 \oplus W^1$ a $Cl(TM)$ -module bundle. Suppose V is an open subset of M with a generalized acyclic real polarization (U, π, D_{fiber}) such that $M \setminus V$ is compact. We will define the

local index $Ind(M, V, W)$ (or $Ind(V, W)$ for short) and show deformation invariance and an excision property. The local index depends on the real polarization on V though we omit it in the notation for simplicity.

Remark 5.1 When $\pi : V \rightarrow U$ is a diffeomorphism, D_{fiber} is a degree-one self-adjoint homomorphism on $W|_V$ anti-commuting with the Clifford multiplication of TV . In this special case the definition of $Ind(V, W)$ is already given in [4, Chapter 3]. (See Definition 3.14 for the setting, Definition 3.21 for the definition in the case of cylindrical end, Theorem 3.20 for deformation invariance, Theorem 3.29 for the excision property, and Section 3.3 for the definition for general case.) We will generalize the argument there.

5.1 Vanishing lemmas

We will show the following lemma later.

Lemma 5.2 *Suppose M is closed and $M = V$ has a generalized acyclic real polarization. Take any Dirac-type operator D on W and write $D = \tilde{D}_U + D_{fiber}$ for some operator \tilde{D}_U . For a real number t , put $D_t = \tilde{D}_U + tD_{fiber}$. Then for any large $t > 0$, $Ker D_t = 0$.*

We also need a slightly generalized version, which is shown later.

Lemma 5.3 *Suppose $M = V$ and M has a cylindrical end with translationally invariant generalized acyclic real polarization on it. Take any Dirac-type operator D on W with translationally invariance on the end, and write $D = \tilde{D}_U + D_{fiber}$ for some operator \tilde{D}_U . For a real number t , put $D_t = \tilde{D}_U + tD_{fiber}$. Then for any large $t > 0$, $Ker D_t \cap \{L^2\text{-sections}\} = 0$.*

Admitting these lemmas we first give the definition and properties of the local index.

5.2 Cylindrical end

We first give the definition for the special case that M has a cylindrical end and every data is translationally invariant on the end.

Lemma 5.4 *Suppose M has a cylindrical end $V = N \times (0, \infty)$ with translationally invariant generalized acyclic real polarization on it. Let ρ be a non-negative smooth cut-off function on M satisfying $\rho = 1$ on $N \times [1, \infty)$ and $\rho = 0$ on $M \setminus V$. For $t > 1$, put $\rho_t := 1 + t\rho$. Take any Dirac-type operator D on W with translationally invariance on the end, and write $D = \tilde{D}_U + D_{fiber}$*

for some operator \tilde{D}_U on the end. Put $D_t = \tilde{D}_U + \rho_t D_{fiber}$ on the end and $D_t = D$ on $M \setminus V$. Then for any large $t > 0$, $Ker D_t \cap \{L^2\text{-sections}\}$ is finite dimensional. Moreover its super-dimension is independent of large t and any other continuous deformations of data.

Proof. From the translationally invariance on the end, we have a generalized acyclic real polarization on $N \times \mathbb{R}/\mathbb{Z}$ and a Dirac-type operator $D_{N \times \mathbb{R}/\mathbb{Z}}$ induced from D_t on $V = N \times (0, \infty)$. Let r be the coordinate of $(0, \infty)$ or \mathbb{R}/\mathbb{Z} . Write $D_{N \times \mathbb{R}/\mathbb{Z}} = \alpha(\partial_r + D_N)$ where α is the Clifford multiplication of ∂_r and D_N is a formally self-adjoint operator on N . From Lemma 5.2, we have $Ker D_{N \times \mathbb{R}/\mathbb{Z}} = 0$ for large t . In particular zero is not an eigenvalue of D_N . It implies that an L^2 -solution f of the equation $D_t f = 0$ on M is exponentially decreasing on the end. Then it is well-known that the space of L^2 -solutions is finite dimensional, and its super-dimension is deformation invariant as far as $Ker D_N = 0$. \square

The super-dimension of the space of L^2 -solutions is equal to the index of $D_t|_{M \setminus V}$ for the Atiyah-Patodi-Singer boundary condition. We use this index as the definition of our local index for the case of cylindrical end.

Definition 5.5 Under the assumption of Lemma 5.4, $Ind(M, V, W)$ is defined to be the super-dimension of $Ker D_t \cap \{L^2\text{-sections}\}$.

The following sum formula follows from a standard argument.

Lemma 5.6 *Suppose $(M, V = N \times (0, \infty), W)$ and $(M', V' = N' \times (0, \infty), W')$ satisfy the assumption of Lemma 5.4. Let N_0 and N'_0 be a connected component of N and N' respectively. Suppose N_0 is isometric to N'_0 via $\phi : N_0 \rightarrow N'_0$, and for some $R > 0$ the map $\phi : N_0 \times (0, R) \rightarrow N'_0 \times (0, R)$ given by $(x, r) \mapsto (\phi(x), R - r)$ can be lifted to the isomorphism between the generalized acyclic real polarizations on them. Then we can glue $M \setminus (N_0 \times [R, \infty))$ and $M' \setminus (N'_0 \times [R, \infty))$ to obtain a new manifold \hat{M} with cylindrical end $\hat{V} = \hat{N} \times (0, \infty)$ for $\hat{N} = (N \setminus N_0) \cup (N' \setminus N'_0)$, and we also have a Clifford module bundle \hat{W} obtained by gluing W and W' on $N_0 \times (0, R) \cong N'_0 \times (0, R)$. Then we have*

$$Ind(\hat{M}, \hat{V}, \hat{W}) = Ind(M, V, W) + Ind(M', V', W').$$

Proof. A proof is given by the APS formula of the indices. More direct proof is also well-known. (For example the above sum formula follows from the excision property [4, Theorem 5.40].) \square

5.3 General case

Now we would like to define the local index for general case.

Let V be an open subset of M with a generalized acyclic real polarization (U, W_U, π, D_{fiber}) such that $M \setminus V$ is compact. We can take a codimension-one closed submanifold N_U of U such that $N = \pi^{-1}(N_U)$ divides M into compact part and non-compact part: For instance, let $f : M \rightarrow [0, \infty)$ be the distance from $M \setminus V$ and define $g : U \rightarrow [0, \infty)$ to be the maximal value of f on the fiber of π . Take a small real number $r > 0$ such that $f^{-1}([0, r])$ is a compact subset of M . Let $h : U \rightarrow [0, \infty)$ be a smooth functions on U satisfying $|h(x) - g(x)| < r/2$ for $x \in U$. Take a regular value r_0 of h satisfying $0 < r_0 < r/2$. Then $N_U = h^{-1}(r_0)$ satisfies the required property.

Let K be the compact part of $M \setminus N$. Note that a neighborhood V_N of N in V is diffeomorphic to $N \times (-\epsilon, \epsilon)$. Then we can construct the Riemannian metric and the translationally invariant generalized acyclic real polarization on (M', V') , where $M' := K \cup (N \times [0, \infty))$ and $V' := (K \cap V) \cup (N \times [0, \infty))$: For instance, let $\phi : M' \rightarrow M$ be a smooth map which is the identity on the complement of $N \times (-\epsilon, \infty)$ and is given by $(x, r) \mapsto (x, \beta(r))$ on $N \times (-\epsilon, \infty)$ for a smooth function β satisfying $\beta(r) = r$ for $-\epsilon < r < -(2/3)\epsilon$, and $\beta(r) = 0$ for $r \geq (1/3)\epsilon$. Define a bundle endmorphism $\tilde{\phi} : TM' \rightarrow TM$ covering ϕ as follows. On the complement of $N \times (-\epsilon, \infty)$, $\tilde{\phi}$ is the identity. On $T(N \times (-\epsilon, \infty)) = TN \times T(-\epsilon, \infty)$, define $\tilde{\phi}$ by $((x, r), (u, v)) \mapsto (\phi(x, r), (u, v))$, where $x \in N$, $r \in (-\epsilon, \infty)$, $u \in T_x N$ and $v \in T_r(-\epsilon, \infty) = \mathbb{R}$. The required deformed Riemannian metric is defined to be the pullback of the original Riemannian metric by $\tilde{\phi}$ as a section of the symmetric tensor product of T^*M . The required deformed Clifford module bundle W' is defined to be the pullback ϕ^*W . Note that we can also construct a Riemannian manifold U' with a cylindrical end $N_U \times (0, \infty)$ and a fiber bundle $\pi' : V' \rightarrow U'$. We define a Dirac type operator D'_{fiber} acting on $\Gamma(W'|_{V'})$ by ϕ^*D_{fiber} . Then (U', π', D'_{fiber}) is a translationally invariant real polarization on (M', V', W') . For this structure the local index is defined by Definition 5.5.

Definition 5.7 We define $Ind(M, V, W)$ to be the local index for (M', V', W') with the translationally invariant real polarization (U', π', D'_{fiber}) .

We have to show the local index is well-defined for the various choice of our construction.

Lemma 5.8 *Suppose we take two codimension-one closed submanifolds N_U and N'_U in U so that M is divided in two ways. Then the local indices defined by these data coincides.*

Proof. Let K and K' be the compact parts of M divided by $N = \pi^{-1}(N_U)$ and $N' = \pi^{-1}(N'_U)$ respectively. Then we can take another N''_U so that the corresponding compact part K'' is contained in the intersection of the interiors of K and K' . It suffices to show that the local indices coincides for K and K'' . Deform the structures on neighborhoods of K and K'' simultaneously to make the structures translationally invariant near K and K'' respectively. Let M_0, M'_0 and \hat{M}_0 be the following manifolds with cylindrical ends.

$$\begin{aligned} M_0 &= K'' \cup (N'' \times [0, \infty)) \\ M'_0 &= (N'' \times (-\infty, 0]) \cup (K \setminus K'') \cup (N \times [0, \infty)) \\ \hat{M}_0 &= K \cup (N \times [0, \infty)) \end{aligned}$$

On the cylindrical ends we have translationally invariant Clifford modules W_0, W'_0 and \hat{W}_0 , and translationally invariant generalized acyclic real polarizations. On M'_0 the generalized acyclic real polarization is given globally. The sum formula of Lemma 5.6 implies $Ind(M_0, W_0) + Ind(M'_0, W'_0) = Ind(\hat{M}_0, \hat{W}_0)$. The vanishing of Lemma 5.3 implies $Ind(M'_0, W'_0) = 0$. Therefore we have $Ind(M_0, W_0) = Ind(\hat{M}_0, \hat{W}_0)$. This is the required equality. \square

5.4 Excision

The well-definedness shown in Lemma 5.8 is the key point for the following formulation of excision property.

Theorem 5.9 (*Excision property*) *Let W be a \mathbb{Z}_2 -graded Clifford module bundle over $Cl(TM)$. Let V be an open subset of M with a generalized acyclic real polarization (U, W_U, π, D_{fiber}) such that $M \setminus V$ is compact. Suppose U' is an open subset of U such that $M' := V' \cup (M \setminus V)$ is an open neighborhood of $M \setminus V$, where we put $V' := \pi^{-1}(U')$. Note that V' has the restricted generalized real polarization. Then we have*

$$Ind(M, V, W) = Ind(M', V', W|_{M'}).$$

Proof. Take a codimension-one submanifold $N_{U'}$ in U' to define $Ind(M', V', W|_{M'})$. Then $N_{U'}$ can be used to define $Ind(M, V, W)$. \square

Proof of Theorem 4.4. We first note that when M is a closed manifold local index $Ind(M, V, W)$ defined by Definition 5.7 is equal to the usual index $Ind(M, W)$ of a Dirac type operator. Under the assumption of Theorem 4.4 from the excision property we have $Ind(M, W) = Ind(\cup_{i=1}^m V_i, W|_{\cup_{i=1}^m V_i})$, which implies the theorem. \square

5.5 Proof of vanishing lemmas

In this subsection we show the vanishing lemmas Lemma 5.2 and Lemma 5.3. Suppose V is an open subset of M with a generalized acyclic real polarization (U, π, D_{fiber}) . Take any order-one formally self-adjoint differential operator \tilde{D} over $W|_V$ with degree one whose principal symbol is given by the composition of the projection $\pi_* : TV \rightarrow TU$ and the Clifford action of TU on $W|_V$. Then $\tilde{D} + D_{fiber}$ is a Dirac-type operator on $W|_V$.

Lemma 5.10 *The anticommutator $D_{fiber}\tilde{D} + \tilde{D}D_{fiber}$ is an order-one differential operator on $W|_V$ which contains only the derivatives along fibers, i.e., it commutes with the multiplication of the pullback of smooth functions on U .*

Proof. Recall that, the principal symbol of \tilde{D} anti-commutes not only with the symbol of D_{fiber} , but also with the whole operator D_{fiber} . The claim follows from this property. It is straightforward to check it using local description. Instead of giving the detail of the local calculation, however, we here give an alternative formal explanation for the above lemma.

For $x \in U$ let \mathcal{W}_x be the sections of the restriction of W on the fiber $\pi^{-1}(x)$. Then $\mathcal{W} = \coprod \mathcal{W}_x$ is formally an infinite dimensional vector bundle over U . We can regard D_{fiber} as an endmorphism on \mathcal{W} . Then D_{fiber} is a order-zero differential operator on \mathcal{W} whose principal symbol is equal to D_{fiber} itself. Then, as a differential operator on \mathcal{W} , $D_{fiber}\tilde{D} + \tilde{D}D_{fiber}$ is an (at most) order-one operator whose principal symbol is given by the anticommutator between the Clifford action by TU and D_{fiber} . This principal symbol vanishes, which implies that the anticommutator is order-zero as a differential operator on \mathcal{W} , i.e., it does not contain derivatives of U -direction. \square

Proof of Lemma 5.2. Let f be a section of W . On each fiber of π at $x \in U$, the second order elliptic operator D_{fiber}^2 is strictly positive. Since $D_{fiber}\tilde{D} + \tilde{D}D_{fiber}$ gives a first order operator on the fiber, a priori estimate implies the estimate

$$\left| \int_{\pi^{-1}(x)} ((D_{fiber}\tilde{D} + \tilde{D}D_{fiber})f, f) \right| \leq C \int_{\pi^{-1}(x)} (D_{fiber}^2 f, f)$$

for some positive constant C . Since M is compact we can take C uniformly.

Therefore we have

$$\begin{aligned}
\int_M ((\tilde{D} + tD_{fiber})^2 f, f) &= \int (\tilde{D}^2 f, f) + t^2 \int (D_{fiber}^2 f, f) \\
&\quad + t \int ((D_{fiber} \tilde{D} + \tilde{D} D_{fiber}) f, f) \\
&\geq \int (\tilde{D}^2 f, f) + (t^2 - Ct) \int (D_{fiber}^2 f, f) \\
&= \int_M |\tilde{D}f|^2 + (t^2 - Ct) \int_M |D_{fiber}f|^2.
\end{aligned}$$

In particular if $t > C$ and $(\tilde{D} + tD_{fiber})f = 0$, then $D_{fiber}f$ is zero, which implies f itself is zero. \square

Proof of Lemma 5.3. The proof is almost identical to the above one for the compact case. What we still need is to guarantee the validity of partial integration. This validity follows from the fact that when $(\tilde{D} + tD_{fiber})f = 0$ and f is in L^2 , then f and any derivative of f decay exponentially on the cylindrical end of V . \square

Note that the exponential decay in the above proof relies on the vanishing lemma for compact case, so we had to separate the proof.

6 2-dimensional case

Let Σ be a compact oriented surface with non-empty boundary $\partial\Sigma$. Let L be a complex line bundle over Σ . Suppose a $U(1)$ -connection is given on the restriction of L on $\partial\Sigma$. When the connection is non-trivial on every boundary component, we can define the local Riemann-Roch number for the data as follows. Fix a product structure $(-\epsilon, 0] \times \partial\Sigma$ on an open neighborhood of the boundary. Then on the collar neighborhood of each connected component of $\partial\Sigma$ the projection onto $(-\epsilon, 0]$ is a circle bundle. Extend the connection smoothly on the neighborhood of the boundary so that we have a flat non-trivial connection on each fiber of the circle bundle. Let $V = V_1$ be the interior of Σ , and V_∞ be the intersection of the open collar neighborhood and V . Then we have the local Riemann-Roch number $RR(V, L)$ (see Theorem 3.3). The deformation invariance of the local Riemann-Roch number implies that it depends only on the initially given data. We often write

$$[\Sigma] = RR(V, L).$$

In this section we calculate $[\Sigma]$ explicitly for several examples (Theorem 6.7). We also show that the local Riemann-Roch number for a non-singular Bohr-Sommerfeld orbit in symplectic case is equal to one (Theorem 6.11). Let us first recall our convention of orientation for boundary. We use the convention for which Stokes theorem holds with positive sign. In other words: Suppose \hat{X} is an oriented manifold, and f is a smooth real function on \hat{X} with 0 a regular value. The orientation of $X = f^{-1}((-\infty, 0])$ and $Y = f^{-1}(0)$ are related to each other as follows. If ω_X and ω_Y are non-vanishing top-degree forms on X and Y compatible with their orientations, then we have $\omega_X|_Y = \rho df \wedge \omega_Y$ for some positive smooth function ρ on Y .

6.1 Type of singularities

6.1.1 BS type singularities

For a positive number ϵ let A_ϵ be the annulus $[-\epsilon, \epsilon] \times S^1$ with the orientation given by $dx \wedge d\theta/2\pi$, where x is the coordinate of $[-\epsilon, \epsilon]$. The projection map $(x, \theta) \mapsto x$ gives a circle bundle structure of A_ϵ . Let L be a complex line bundle over A_ϵ , and ∇ a $U(1)$ -connection on L . Let $e^{\sqrt{-1}h(x)}$ be the holonomy along the circle of $\{x\} \times S^1$ for the orientation given by $d\theta$. Suppose $h(x)$ is continuous and $h(0) = 0$.

Definition 6.1 (positive/negative BS) When $h(x) > 0$ for $x > 0$ and $h(x) < 0$ for $x < 0$, we call the fiber at 0 a *positive BS type*. When $h(x) < 0$ for $x > 0$ and $h(x) > 0$ for $x < 0$, we call the fiber at 0 a *negative BS type*. See Figure 1.

6.1.2 Disk type singularities

Let D^2 be an oriented disk. Choose a polar coordinate r and θ so that D^2 becomes a unit disk and the orientation of D^2 is compatible with $dr \wedge d\theta$ outside the origin. In particular the orientation of the boundary ∂D^2 is compatible with $d\theta$. The projection map $(r, \theta) \mapsto r$ gives a circle bundle structure on neighborhood of the boundary of D^2 .

Let L be a complex line bundle over D^2 , and ∇ a $U(1)$ -connection on L . Let $e^{\sqrt{-1}h(r)}$ be the holonomy along the circle of radius r centered in the origin. The orientation of the circle is defined as the boundary of the disk of radius r centered in the origin. We can take $h(r)$ continuous with limit value $\lim_{r \rightarrow 0} h(r) = 0$.

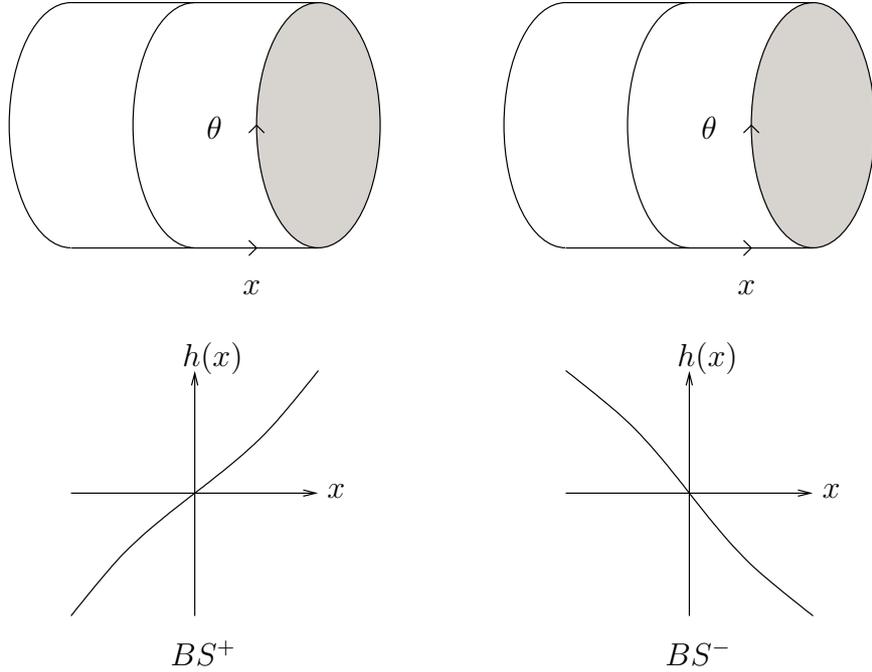


Figure 1: positive/negative BS

Definition 6.2 (positive/negative disk) When $h(r) > 0$ for small $r > 0$ we call the singularity positive-disk type. When $h(r) < 0$, then we call it negative-disk type. See Figure 2.

6.1.3 Trinion type singularities

Let Σ be a genus 0 oriented surface with three holes, i.e, Σ is a trinion. For each boundary component its collar is diffeomorphic to the product of a circle and an interval. The projection onto the interval gives a circle bundle structure near the boundary.

Let L be a complex line bundle over Σ , and ∇ a $U(1)$ -connection on L . Suppose ∇ is flat and its holonomy along each component of the boundary of Σ is non-trivial. Let $e^{\sqrt{-1}h_k}$ ($k = 1, 2, 3$) be the three holonomies along the three components of the boundary of Σ . The orientations of the boundary components are defined as the boundary of the oriented manifold Σ . Then the product of these three holonomies is equal to 1. We can take h_k satisfying $0 < h_k < 2\pi$.

From our assumption, there is no Bohr-Sommerfeld orbit in the collar neighborhood of the boundary.

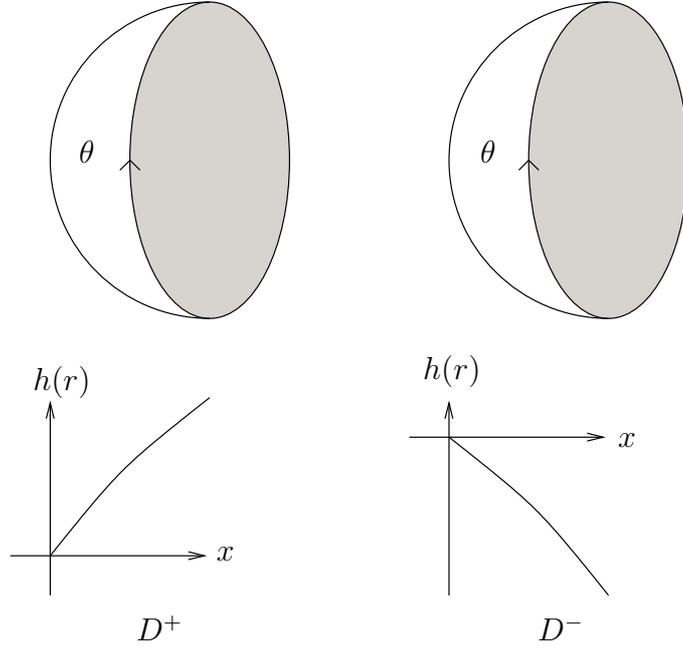


Figure 2: positive/negative disk

Definition 6.3 (small/large trinion) When $h_1 + h_2 + h_3 = 2\pi$, we call Σ a small trinion. When $h_1 + h_2 + h_3 = 4\pi$, we call Σ a large trinion.

6.2 Examples

Example 6.4 (a torus over a circle with degree n line bundle) Let x and y the coordinate of \mathbb{R}^2 . Let $\hat{\nabla}$ be the $U(1)$ -connection on the trivial complex line bundle over \mathbb{R}^2 with connection form $-\sqrt{-1}xdy$.

1. The curvature $F_{\hat{\nabla}}$ of $\hat{\nabla}$ is $-\sqrt{-1}dx \wedge dy$. In particular

$$\frac{\sqrt{-1}}{2\pi}F_{\hat{\nabla}} = \frac{1}{2\pi}dx \wedge dy$$

2. The holonomy along the straight line from the point $(x, 0)$ to (x, y) is equal to $\exp(\sqrt{-1}xy)$.
3. The connection is invariant under the action of $(m, 2\pi n) \in \mathbb{Z} \oplus 2\pi\mathbb{Z}$ given by

$$(x, y, u) \mapsto (x + m, y + 2\pi n, e^{\sqrt{-1}my}u).$$

For a positive integer N let T_N^2 be the quotient of \mathbb{R}^2 divided by the subgroup $N\mathbb{Z} \oplus 2\pi\mathbb{Z}$, on which we have the quotient complex line bundle L_N with the quotient connection ∇_N . Then $(\sqrt{-1}/2\pi)F_{\nabla_N}$ gives a symplectic structure on T^2 . The projection onto the first factor $T_N^2 \rightarrow \mathbb{R}/N\mathbb{Z}$ is a Lagrangian fibration. The holonomy along the fiber at $x \bmod N$ is $e^{2\pi\sqrt{-1}x}$. The Bohr-Sommerfeld orbits are the fibers at $x \in \mathbb{Z}/N\mathbb{Z}$, and all of them are positive BS singularities. The degree of L_N is equal to N and the Riemann-Roch number is N from the Riemann-Roch theorem and it is equal to the number of positive BS singularities.

Example 6.5 (a sphere with degree-zero line bundle) Let D^+ and D^- be disks, and L^+ and L^- complex line bundles over D^+ and D^- with $U(1)$ -connections such that the connections are flat near the boundaries, respectively. Suppose D^+ and D^- are a positive disk and a negative disk respectively, and the holonomies along the boundaries are $e^{\sqrt{-1}\epsilon}$ and $e^{-\sqrt{-1}\epsilon}$ for a small positive number ϵ . Patch (D^+, L^+) and (D^-, L^-) together to obtain a complex line bundle over an oriented sphere with a $U(1)$ -connection. Since the degree of the $U(1)$ -bundle is zero, the Riemann-Roch number is 1.

Example 6.6 (a surface with a trinion decomposition with a flat line bundle)

Let ϵ be a small positive number. Let P^S be a trinion with a flat $U(1)$ -bundle whose holonomies along the boundary components are $e^{\sqrt{-1}(\pi-\epsilon)}$, $e^{\sqrt{-1}(\pi-\epsilon)}$, and $e^{\sqrt{-1}2\epsilon}$. Let P^L be a trinion with a flat $U(1)$ -connection whose holonomies along the boundary components are $e^{\sqrt{-1}(\pi+\epsilon)}$, $e^{\sqrt{-1}(\pi+\epsilon)}$, and $e^{-\sqrt{-1}2\epsilon}$. Then P^S is a small trinion and P^L is a large trinion. For an integer $g \geq 2$, take $(g-1)$ copies of P^S and $(g-1)$ -copies of P^L , and patch them together to obtain a flat connection on a closed oriented surface with genus g . The Riemann-Roch number is $1-g$.

6.3 Local Riemann-Roch numbers

Let $[BS^+]$ and $[BS^-]$ be the contribution of a positive and negative BS respectively. Let $[D^+]$ and $[D^-]$ be the contribution of a positive and negative disk respectively. Let $[P^S]$ and $[P^L]$ be the contribution of a small and large trinion respectively.

Theorem 6.7

$$[BS^+] = 1, \quad [BS^-] = -1, \quad [D^+] = 1, \quad [D^-] = 0, \quad [P^S] = 0, \quad [P^L] = -1$$

Proof. These are consequences of Lemma 6.8 and Lemma 6.9 below. \square

Lemma 6.8 $[P^L] + [BS^+] = [P^S]$, $[BS^-] + [BS^+] = 0$, $[D^-] + [BS^+] = [D^+]$.

Proof. The three relations are shown in a similar way. We just show the first relation. Let ϵ be a small positive number. Let P^S be a trinion with a flat $U(1)$ -bundle whose holonomies along the boundary components are $e^{\sqrt{-1}(\pi-\epsilon)}$, $e^{\sqrt{-1}(\pi-\epsilon)}$, and $e^{\sqrt{-1}2\epsilon}$. Let P^L be a trinion with a flat $U(1)$ -connection whose holonomies along the boundary components are $e^{\sqrt{-1}(\pi+\epsilon)}$, $e^{\sqrt{-1}(\pi+\epsilon)}$, and $e^{-\sqrt{-1}2\epsilon}$. Then P^S is a small trinion and P^L is a large trinion. Let A be an oriented annulus with $U(1)$ -connection such that the connection is flat near the boundary. Suppose A is positive-BS type and both the holonomies of the two boundary components are $e^{\sqrt{-1}2\epsilon}$ for the orientation as boundary of A . Patch P^L and A together along the boundary components with holonomies $e^{-\sqrt{-1}2\epsilon}$ and $e^{\sqrt{-1}2\epsilon}$ to obtain another trinion with a $U(1)$ -connection. The glued $U(1)$ -connection can be deformed to a flat $U(1)$ -connection isomorphic to the one on P^S without changing the connection near boundary components. \square

Lemma 6.9 $[BS^+] = 1$, $[P^S] + [P^L] = -1$, $[D^+] + [D^-] = 1$.

Proof. The three relations are consequences of Example 6.4, Example 6.5, and Example 6.6 respectively. \square

Remark 6.10 It is possible to show $[BS^+] = 1$ directly without appealing the Riemann-Roch theorem in the following way. Let $L = \mathbb{R} \times S^1 \times \mathbb{C}$ be the trivial complex line bundle on $\mathbb{R} \times S^1$. For $0 < \delta < 1$ let $h(x)$ be a smooth increasing function on \mathbb{R} with $h(0) = 0$, $h(x) \equiv \delta$ for sufficiently large x and $h(x) \equiv -\delta$ for sufficiently small x . We put $\rho(x) := h(x)/2\pi$. Consider the $U(1)$ -connection on L of the form $\nabla = d - \sqrt{-1}\rho(x)d\theta$.

For a L^2 -section s of L , we first solve the equation

$$0 = \bar{\partial}_L s = \bar{\partial} s + \rho(x)s. \quad (1)$$

By taking the Fourier expansion of s with respect to θ , s is written as

$$s = \sum_{n \in \mathbb{Z}} a_n(x) e^{nz},$$

where $z = x + \sqrt{-1}\theta$. Then, s satisfies (1) if and only if each a_n is of the form

$$a_n = c_n \exp\left(-\int_0^x \rho(x) dx\right)$$

for some constant c_n . Since $h(x) \equiv \pm\delta$ for sufficiently large, or small x and since s is a L^2 -section, it is easy to see that $c_n = 0$ except for $n = 0$. This implies that the kernel of $\bar{\partial}_L$ is one-dimensional.

Next we solve the equation

$$0 = \bar{\partial}_L^* s = \bar{\partial} s - \rho(x)s. \quad (2)$$

By the similar argument we can show that $c_n = 0$ for all $n \in \mathbb{Z}$. This implies that the kernel of $\bar{\partial}_L^*$ is zero-dimensional. Thus, $[BS^+] = 1$.

6.4 Higher dimensional Bohr-Sommerfeld orbits

We show:

Theorem 6.11 *In symplectic formulation, the local Riemann-Roch number of a non-singular Bohr-Sommerfeld orbit is one.*

Proof. It is known that the neighborhoods of two Bohr-Sommerfeld orbits are isomorphic to each other together with prequantum line bundle with connection: Recall that the fibers in a neighborhood of a Bohr-Sommerfeld orbit are parameterized by their periods. If we fix a local Lagrangian section, and a trivialization of the first homology group of the local fibers, then we can write down a canonical coordinate. Therefore it suffices to give one example for which the claim is satisfied. An example of a Lagrangian fibration with exactly one n -dimensional Bohr-Sommerfeld orbit is given by the product of n -copies of the fiber bundle structure of the torus T_N^2 for $N = 1$ in Example 6.4. In this case our convention of the orientation for the symplectic manifold coincides with the product orientation. The Riemann-Roch number is equal to one because it is equal to the n -th power of $RR(T_1^2) = 1$. \square

Remark 6.12 It would be expected that, if we use appropriate boundary condition, then it would be possible to define a local Riemann-Roch number for the product $D^+ \times P^L$, and moreover it would be equal to the product $[D^+][P^L]$, i.e., -1 . A crucial problem here is that there is no Lagrangian fibration structure on the whole neighborhood of the boundary of $D^+ \times P^L$. In fact it is possible to extend our formulation to such cases. We will discuss this elsewhere [3].

7 Comments

It is possible to extend our construction for various situations.

1. Isotropic fibrations: When we have a integrable system which is not necessary complete, if all the orbits are "periodic" and form tori, we can extend our argument. It would be an interesting problem to investigate the case when the orbits are not periodic.
2. Manifolds with boundaries and corners: Our definition of the local index is related to manifolds with boundaries. For manifolds with coners, it is possible to extend our construction.
3. Equivariant and family version: Our constructin is natural, so if a compact Lie group acts and preserves the data, then everything is formulated equivariantly. Similarly we have a family version of our construction.
4. Equivariant mod-2 indices: A modification of the localization property explained in this paper can be applied to define *G-equivariant mod-2 indices* valued in $R(G)/RO(G)$ or $R(G)/RSp(G)$ for even dimensional G -Spin^c-manifolds with G -spin structures on its end [6].

We will discuss them elsewhere [3].

A Spin^c-structures

In this appendix we recall our convention for spin^c-strctures on oriented manifolds. (See also [4, the end of §2.3].) A spin^c-structure is usually defined for an oriented Riemannian manifold. In this paper we take a convention of spin^c-structures which do not need any Riemannian metrics. In fact a spin^c-structure itself is defined at the principal bundle level as follows. Let $GL_m^+(\mathbb{R})$ be the group of orientation preserving linear automorphisms of \mathbb{R}^m . Since $GL_m^+(\mathbb{R})$ has the same homotopy type as that of $SO_m(\mathbb{R})$, there is a unique non-trivial double covering of $GL_m^+(\mathbb{R})$. We denote it by $p : \widetilde{GL}_m^+(\mathbb{R}) \rightarrow GL_m^+(\mathbb{R})$. Consider the diagonal action of $\mathbb{Z}/2\mathbb{Z}$ on $\widetilde{GL}_m^+(\mathbb{R}) \times \mathbb{C}^\times$, where the action on the first factor is the deck transformation and the action on the second factor is defined by $z \mapsto -z$. Let $\widetilde{GL}_m^+(\mathbb{R}) \times_{\mathbb{Z}/2\mathbb{Z}} \mathbb{C}^\times$ be the quotient group by this diagonal action. Note that there is a canonical homomorphism $\hat{p} : \widetilde{GL}_m^+(\mathbb{R}) \times_{\mathbb{Z}/2\mathbb{Z}} \mathbb{C}^\times \rightarrow GL_m^+(\mathbb{R})$ defined by

$$\hat{p} : [\tilde{g}, z] \mapsto p(\tilde{g}).$$

Definition A.1 Let M be an oriented manifold and P_M the associated frame bundle over M , which is a principal $GL_m^+(\mathbb{R})$ -bundle. A *spin^c-structure* on M is a pair (\tilde{P}_M, q_M) satisfying the following two conditions.

1. \tilde{P}_M is a principal $\widetilde{GL}_m^+(\mathbb{R}) \times_{\mathbb{Z}/2\mathbb{Z}} \mathbb{C}^\times$ -bundle over M .
2. q_M is a bundle map from \tilde{P}_M to P_M which is equivariant with respect to the canonical homomorphism \hat{p} .

Remark A.2 Though a spin^c -structure can be defined without any Riemannian metric, we need to fix a Riemannian metric to define a Clifford module bundle over an oriented manifold.

Remark A.3 The natural embedding $GL_n(\mathbb{C}) \hookrightarrow GL_{2n}^+(\mathbb{R})$ induces a homomorphism $GL_n(\mathbb{C}) \rightarrow \widetilde{GL}_{2n}^+(\mathbb{R}) \times_{\mathbb{Z}/2} \mathbb{C}^\times$. This means that an almost complex manifold has a canonical spin^c -structure in our convention.

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